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AN

ELEMENTARY ARITHMETIC,

DESIGNED FOR

ACADEMIES AND SCHOOLS;

ALSO,

SERVING AS AN INTRODUCTION TO THE

HIGHER ARITHMETIC.

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PREFACE.

This Elementary Work on Arithmetic, which may be regarded as an introduction to my Higher Arithmetic, contains as full a treatise on this science, as is usually given in our School Arithmetics.

I have endeavored to adapt it to the present wants of our Schools and Academies. I have excluded Mensuration, Mechanical Powers, Bookkeeping, &c., as subjects which are foreign to an Elementary Arithmetic.

It is believed that there are many things which, to an experienced Teacher, will be considered as improvements. To point out all the particulars in which this work differs from other similar works, would be tedious; still I can not help mentioning two points which I consider of considerable importance. The first is, that I have been careful to treat of Decimal Fractions, before treating of Federal Money. After the student has become familiar with the nature of Decimal Fractions, he can find no difficulty in operating upon Federal money, since the cents and mills may be regarded as decimals of a dollar; this I consider much better than to treat Federal Money as Denominate Numbers.

The second thing which I would refer to, is the method of Extracting the Cube Root of a number. It is now more than twenty years since Mr. Horner gave a new method

of solving numerical equations from which the Arithmetical Rule for extracting the cube root has been deduced. It is surprising that this new method has not been more extensively made known; but from its simplicity, as well as brevity, it can not fail of soon taking the place of all other Arithmetical Rules for determining the cube root of a number.

I have not given any questions which would require rules or principles which are not strictly arithmetical. In all cases I have thought it best to give the answers; and such questions as I have feared might be too difficult for beginners, I have given at length, what I deemed the simplest method of solution.

GEO. R. PERKINS.

UTICA, April, 1844.

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ARITHMETIC.

Article 1. Arithmetic is the science of numbers.

The operations of arithmetic are carried on by the aid of five distinct rules, viz.: *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*. These are usually called the Fundamental Rules of arithmetic, because all other rules are founded upon them.

What is Arithmetic? How many distinct rules has it for its operations? Repeat their names. What are these usually called? Why are they so called?

NUMERATION.

2. Numeration explains the method of numbering.

Various methods of notation and numeration were used by the ancients; we shall content ourselves with mentioning two, the common or *Arabic* method, and the *Roman* method.

In the common method ten characters are employed.

These characters when written are,

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

When printed they become,

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

They have the following names:

1 is called One, or one Unit,
2 is called Two, or two Units,
3 is called Three, or three Units,
4 is called Four, or four Units,
5 is called Five, or five Units,
6 is called Six, or six Units,
7 is called Seven, or seven Units,
8 is called Eight, or eight Units,
9 is called Nine, or nine Units,
0 is called Naught, Cipher, or Zero.

Each of these characters, except *zero*, is called a *digit*; and the first nine, when taken together, are called the *nine digits*.

Any digit is called a *significant* figure.

What is numeration? How is the common method sometimes called? In this method how many characters are employed? What are the names of these characters? Which are called digits? What is a significant figure?

3. Figures have two distinct values, called *simple* and *local*.

The *simple* value of a figure is its value when no reference is made to other figures. The simple value is always the same.

The *local* value of a figure depends wholly upon the situation it occupies in reference to other figures with which it is connected.

We have already said that the character 0, by itself, has no value; that is, it has no simple value; but when placed at the right of a significant figure, it causes it to represent ten times its simple value. Thus:

40 is the same as Forty, or ten times 4,
70 is the same as Seventy, or ten times 7,
80 is the same as Eighty, or ten times 8.

In the above numbers, 4, 7, and 8, are said to occupy the place of tens, since they represent ten times their simple values.

If two zeros are placed at the right of a significant figure, its value becomes increased a hundred fold. Thus:

400 is the same as Four hundred, or one hundred times 4.

700 is the same as Seven hundred, or one hundred times 7.

Three zeros placed at the right of a figure causes its simple value to become increased a thousand fold. Thus:

4000 is the same as Four thousand, or one thousand times 4.

In the same way we might proceed for a greater number of zeros; observing that every additional zero increases the preceding value tenfold.

When several significant figures are connected together, the right-hand figure has only its simple value; the second figure, counting from the right, has ten times its simple value; the third figure has one hundred times its simple value; the fourth figure has one thousand times its simple value, and so on; each figure increasing in a tenfold ratio as we pass from the right toward the left. Thus, in the number 3456, the figure 6 has only its simple value, 5 has ten times its simple value, and is therefore the same as 50; 4 has one hundred times its simple value, and is therefore the same as 400; 3 has one thousand times its simple value, and is therefore the same as 3000. Hence the above number is *three thousand four hundred and fifty-six*.

The number 20406 consists of 6 units, 0 tens, 4 hundreds, 0 thousands, and 2 ten thousands. It is read *twenty thousand four hundred and six*.

How many distinct values have figures? What is the simple value of a figure? Upon what does the local value depend? What change is made upon a figure by placing one zero at the right of it? What change when two zeros are thus placed? Every additional zero increases the value in what ratio? When several significant figures are connected together, which figure has only its simple value?

4. We shall not find it difficult to read any number not consisting of more than nine places of figures, if we carefully study the following

NUMERATION TABLE.

9 Hundreds of Millions.	8 Tens of Millions.	7 Millions.	6 Hundreds of Thous.	5 Tens of Thousands.	4 Thousands.	3 Hundreds.	2 Tens.	1 Units.	Places, or order.
.	.	.	.	.	.	.	.	3	
.	.	.	.	.	.	.	5	4	
.	.	.	.	.	.	2	6	7	
.	.	.	.	.	7	4	3	8	
.	.	.	.	7	2	9	1	2	
.	.	.	4	3	2	6	5	4	
.	.	6	3	5	7	4	2	1	
.	7	6	8	2	4	6	9	8	
6	8	5	7	3	1	9	7	5	

The names, Units, Tens, Hundreds, &c., placed at the head of this table, should be committed to memory.

The numbers in this table are numerated, and read as follows: Units—*three*. Units, tens—*fifty-four*. Units, tens, hundreds—*two hundred and sixty-seven*. Units, tens, hundreds, thousands—*seven thousand four hundred and thirty-eight*, &c.

EXAMPLES.

Numerate and read the following numbers:

4
3 6
2 0 5
1 2 3 7
2 7 8 3 5
1 0 2 0 0 7
6 3 0 0 0 6 9
5 4 1 2 8 9 0 0
6 5 1 2 3 4 5 6 7

Also, write down the following numbers under each other, so that units may stand under units, tens under tens, hundreds under hundreds, &c.

Seventy-three.

Three hundred and thirty-seven.

Eight thousand six hundred and one.

Ninety-seven thousand three hundred and forty-three.

Three hundred thousand five hundred and eleven.

Six millions one thousand and twenty-five.

Forty-three millions and seventeen.

Two hundred and thirty-three millions and ten thousand.

5. Thus far we have shown how to numerate and read numbers which do not contain more than nine places of figures. When there are more than nine places of figures, it will be convenient to divide them into periods of three figures each, as in the following

TABLE.

Period	Name	Place		Figure
&c.	&c.	&c.	&c.	&c.
9th Period.	Septillions.	Hundreds of Septillions.	27	6
		Tens of Septillions.	26	5
		Septillions.	25	7
8th Period.	Sextillions.	Hundreds of Sextillions.	24	8
		Tens of Sextillions.	23	9
		Sextillions.	22	3
7th Period.	Quintillions.	Hundreds of Quintillions.	21	4
		Tens of Quintillions.	20	2
		Quintillions.	19	1
6th Period.	Quadrillions.	Hundreds of Quadrillions.	18	7
		Tens of Quadrillions.	17	5
		Quadrillions.	16	3
5th Period.	Trillions.	Hundreds of Trillions.	15	2
		Tens of Trillions.	14	1
		Trillions.	13	6
4th Period.	Billions.	Hundreds of Billions.	12	4
		Tens of Billions.	11	8
		Billions.	10	5
3d Period.	Millions.	Hundreds of Millions.	9	3
		Tens of Millions.	8	2
		Millions.	7	7
2d Period.	Thousands.	Hundreds of Thousands.	6	4
		Tens of Thousands.	5	8
		Thousands.	4	9
1st Period.	Units.	Hundreds.	3	4
		Tens.	2	3
		Units.	1	2

By this table we discover that each period, or group of three figures, takes a new name, by which means the numeration of all numbers is made to depend upon that of three figures.

6. The above method of numerating, by giving to each period of three figures an independent name, is due to the *French*. There is another method, sometimes used, called the *English* method. It consists in giving a new name to each period of six figures. The *French* way is the simplest, and is becoming generally adopted. We will exhibit the two methods at one view in the following

TABLE.

French Method.		*English Method.*
Hundreds of Octillions.	3	Hunds. of Thous. of Quadrillions.
Tens of Octillions.	3	Tens of Thous. of Quadrillions.
Octillions.	3	Thousands of Quadrillions.
Hundreds of Septillions.	3	Hundreds of Quadrillions.
Tens of Septillions.	3	Tens of Quadrillions.
Septillions.	3	Quadrillions.
Hundreds of Sextillions.	3	Hunds. of Thous. of Trillions.
Tens of Sextillions.	3	Tens of Thousands of Trillions.
Sextillions.	3	Thousands of Trillions.
Hundreds of Quintillions.	3	Hundreds of Trillions.
Tens of Quintillions.	3	Tens of Trillions.
Quintillions.	3	Trillions.
Hundreds of Quadrillions.	3	Hunds. of Thous. of Billions.
Tens of Quadrillions.	3	Tens of Thousands of Billions.
Quadrillions.	3	Thousands of Billions.
Hundreds of Trillions.	3	Hundreds of Billions.
Tens of Trillions.	3	Tens of Billions.
Trillions.	3	Billions.
Hundreds of Billions.	3	Hunds. of Thous. of Millions.
Tens of Billions.	3	Tens of Thousands of Millions.
Billions.	3	Thousands of Millions.
Hundreds of Millions.	3	Hundreds of Millions.
Tens of Millions.	3	Tens of Millions.
Millions.	3	Millions.
Hundreds of Thousands.	3	Hundreds of Thousands.
Tens of Thousands.	3	Tens of Thousands.
Thousands.	3	Thousands.
Hundreds.	3	Hundreds.
Tens.	3	Tens.
Units.	3	Units.

By the French method of numerating, how many figures are connected in a period? How many do the English connect in a period? Which method is to be preferred?

7. After the student has carefully examined this table, let him be required to numerate and read, by dividing into periods of three figures, the following numbers:

1347835674116
3478567321752005
7545627832700571 7
63345626748913654 5
4565421340010020543 7
4677434869217854121234564 89

Let him also separate them into periods of six figures, according to the English method, and then numerate and read them.

ROMAN NOTATION.

8. The Romans, as well as many other nations, expressed numbers by certain letters of the alphabet. The Romans made use of only seven capital letters, viz.: I for *one;* V for *five;* X for *ten;* L for *fifty*; C for *one hundred;* D for *five hundred;* M for *one thousand.* The other numbers they expressed by various repetitions and combinations of these letters, as in the following

TABLE.

1	expressed	by	I.
2	"	"	II.
3	"	"	III.
4	"	"	IV. or IIII.
5	"	"	V.
6	"	"	VI.
7	"	"	VII.
8	"	"	VIII.
9	"	"	IX.
10	"	"	X.
50	"	"	L.
100	"	"	C.
500	"	"	D.
1000	"	"	M.
2000	"	"	MM.
5000	"	"	$\overline{V}$

As often as any character is repeated, so many times is its value repeated.

A less character before a greater, diminishes its value. A less character after a greater, increases its value.

A bar (—) over any number, increases it 1000 fold.

By what means did the Romans express numbers? In this notation, how did repeating a letter affect the value which it represented? How was the value of a character affected when one of less value was placed before it? How when a character of less value was placed after it? How was the value affected by a bar drawn over it?

ADDITION OF SIMPLE NUMBERS

9. Simple Addition is putting together several numbers of the same kind or denomination.

The sum total which is obtained by adding several numbers together, is called the amount.

Before explaining the method of adding numbers, we will show the use of the two symbols $=$, $+$.

The symbol $=$, is called the sign of equality, and when placed between two quantities, it indicates that they are equal. Thus $1=100 cents, implies that one dollar is equal to one hundred cents.

The symbol $+$, is called the sign of addition, and when placed between two quantities, indicates that those quantities are to be added. Thus $3+4=7$, denotes that the sum of 3 and 4 is equal to 7.

What is simple addition? What is the result obtained by adding several numbers together, called? Describe the symbol of equality. Describe that of addition.

By the assistance of these two symbols we may form the following

ADDITION TABLE.

$2+0=2$	$3+0=3$	$4+0=4$	$5+0=5$
$2+1=3$	$3+1=4$	$4+1=5$	$5+1=6$
$2+2=4$	$3+2=5$	$4+2=6$	$5+2=7$
$2+3=5$	$3+3=6$	$4+3=7$	$5+3=8$
$2+4=6$	$3+4=7$	$4+4=8$	$5+4=9$
$2+5=7$	$3+5=8$	$4+5=9$	$5+5=10$
$2+6=8$	$3+6=9$	$4+6=10$	$5+6=11$
$2+7=9$	$3+7=10$	$4+7=11$	$5+7=12$
$2+8=10$	$3+8=11$	$4+8=12$	$5+8=13$
$2+9=11$	$3+9=12$	$4+9=13$	$5+9=14$

6+0= 6	7+0= 7	8+0= 8	9+0= 9
6+1= 7	7+1= 8	8+1= 9	9+1=10
6+2= 8	7+2= 9	8+2=10	9+2=11
6+3= 9	7+3=10	8+3=11	9+3=12
6+4=10	7+4=11	8+4=12	9+4=13
6+5=11	7+5=12	8+5=13	9+5=14
6+6=12	7+6=13	8+6=14	9+6=15
6+7=13	7+7=14	8+7=15	9+7=16
6+8=14	7+8=15	8+8=16	9+8=17
6+9=15	7+9=16	8+9=17	9+9=18

Let the student be required to answer the following questions:

4+3=how many?
2+5+1=how many?
5+6+7+2=how many?
8+9+2+1+7=how many?
6+7+5+4+3+2=how many?
1+2+4+3+5+7+6=how many?

10. From what has been explained under notation, we know, that ten units is equal to one ten, ten tens is equal to one hundred, ten hundreds is equal to one thousand, and so on; ten of any order is equal to one of the next superior order. Hence, for adding numbers of the same denomination, we deduce this

RULE.

I. Place the numbers to be added under each other, so that units may be under units, tens under tens, hundreds under hundreds, and so on for the higher orders.

II. Commencing at the right, find the sum of the digits in the column of units; if this sum is less than ten, place it immediately under the unit column; but if it is not less than ten, see how many tens it contains, and how many units over; write down the number expressing the units, and carry the number expressing the tens to the sum in the next column. In this way proceed with each column, observing to carry as in the first case, that is, one for every ten. When we reach the last column, the whole amount must be set down.

How do you write the numbers for addition? Where do you commence to add? If the sum is expressed by a single digit, how do you dispose of it? When it is not less than ten, how do you proceed? What is the rule with regard to carrying? How do you do when you come to the last column?

EXAMPLES.

1. What is the sum of the following numbers: 3758, 4903, 7006, 3713, and 3781?

In this example, after placing the numbers under each other, agreeable to the first part of the rule, we find the sum of the numbers in the column of units to be 21, which consists of 2 tens and 1 unit; we write down the 1 and carry 2 to the next column, by which means its sum becomes 16; we now write down the 6 and carry 1 to the next column, which thus becomes 31; we write the 1 and carry 3 to the next column, by which means we find 23; this being the last column, we write the whole amount.

OPERATION.
3758
4903
7006
3713
3781
23161 Sum.

(2.)	(3.)	(4.)
3734	56430	7921341
4113	12798	82345768
3221	34457	79013265
4560	21325	7890275
15628 Sum.	125010 Sum.	177170649 Sum.

PROOF OF ADDITION.

11. The method of proving, or testing the work of addition, is generally to commence at the top of the respective columns and add downwards, carrying one for every ten as before; if the sum is the same as when the columns were added upwards, the work is then supposed to be correct. This proof is not infallible, since mistakes may occur in both operations.

How is the work of addition generally proved? Is this method of proof infallible? Why not?

(5.)	(6.)	(7.)
34567890	43345678	123423434
2357911	21123355	23785432
234567	27893	9876543
24897	54689	751002
64	734321	10200
37185329	65285936	157846611

8. Add 123405, 2354210, 794327, and 36547, together. *Ans.* 3308489.

9. Add 275602, 345607, 4567801, and 365, together. *Ans.* 5189375.

10. Add 100375, 406780, 4673005, 4112, and 2478, together. *Ans.* 5186750.

11. Add 1034001, 78954, 379205, 367001, and 45637, together. *Ans.* 1904798.

12. What is the sum of the following numbers: Three thousand six hundred and fifty, seven thousand eight hundred and thirty-two, eleven thousand five hundred and sixty-seven, ten thousand and fifty-six, four hundred and seventy-two? *Ans.* 33577.

13. What is the sum of the numbers, four thousand three hundred and seventy-three, three thousand one hundred and fourteen, one thousand two hundred and twenty-three, six hundred and fifty-four. *Ans.* 9364.

14. Find the number of days in a year, the days of the respective months being as follows: January 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, December 31. *Ans.* 365.

15. A man drew five loads of brick; in the first load he had 1209, in the second load 1453, in the third load 1101, in the fourth load 1212, and in the fifth load 1303. How many brick were there in all? *Ans.* 6278.

16. If there is shipped from the United States, 15624 barrels of flour to Sweden, 250 barrels to Holland, 205154 barrels to England, 6401 to Texas, 19602 to Mexico, what is the whole amount? *Ans.* 247031.

17 In 1837 the United States exported 100232 hogs-

heads of tobacco, in 1838 they exported 100592, in 1839 they exported 78995, in 1840 they exported 119484, in 1841 they exported 147828. How many hogsheads of tobacco were exported during these five years?

Ans. 547131.

18. If the cotton crop of the United States is estimated at 1360532 bales for the year 1839, 2177835 bales for the year 1840, 1634945 bales for the year 1841, and 1683574 bales for the year 1842, how many bales will the four years' crops amount to? *Ans.* 6856886.

19. In 1839 the Onondaga Springs produced 2864718 bushels of salt, in 1840 they produced 2622305 bushels, in 1841 they produced 3340769 bushels, in 1842 they produced 2291903 bushels. What is the whole number of bushels during the above four years?

Ans. 11119695.

20. The United States exported in bullion and specie, in 1838, 3508046 dollars; in 1839, 8776743 dollars; in 1840, 8417014 dollars; in 1841, 10034332 dollars. How much was exported during these four years?

Ans. 30736135 dollars.

SUBTRACTION OF SIMPLE NUMBERS.

12. SUBTRACTION is taking a less number from a greater.

The greater number is called the *minuend*, and the smaller number is called the *subtrahend;* the result is called the *remainder* or *difference.*

The symbol for subtraction is —. When this symbol is placed between two numbers, it indicates that the second is to be subtracted from the first. Thus, 8—5, denotes that 5 is to be taken from 8. The remainder being 3, we have 8—5=3.

What is subtraction? What is the greater number called? What is the smaller number called? What is the result called? What symbol is used to denote subtraction?

By using this symbol we may form the following

SUBTRACTION TABLE.

2−2= 0	3−3= 0	4−4= 0	5−5= 0
3−2= 1	4−3= 1	5−4= 1	6−5= 1
4−2= 2	5−3= 2	6−4= 2	7−5= 2
5−2= 3	6−3= 3	7−4= 3	8−5= 3
6−2= 4	7−3= 4	8−4= 4	9−5= 4
7−2= 5	8−3= 5	9−4= 5	10−5= 5
8−2= 6	9−3= 6	10−4= 6	11−5= 6
9−2= 7	10−3= 7	11−4= 7	12−5= 7
10−2= 8	11−3= 8	12−4= 8	13−5= 8
11−2= 9	12−3= 9	13−4= 9	14−5= 9
6−6= 0	7−7= 0	8−8= 0	9−9= 0
7−6= 1	8−7= 1	9−8= 1	10−9= 1
8−6= 2	9−7= 2	10−8= 2	11−9= 2
9−6= 3	10−7= 3	11−8= 3	12−9= 3
10−6= 4	11−7= 4	12−8= 4	13−9= 4
11−6= 5	12−7= 5	13−8= 5	14−9= 5
12−6= 6	13−7= 6	14−8= 6	15−9= 6
13−6= 7	14−7= 7	15−8= 7	16−9= 7
14−6= 8	15−7= 8	16−8= 8	17−9= 8
15−6= 9	16−7= 9	17−8= 9	18−9= 9

Let the student be required to answer the following questions:

8− 2=how many?
11− 2=how many?
8− 3=how many?
10− 3=how many?
12− 3=how many?
7− 4=how many?
9− 4=how many?
11− 4=how many?
13− 4=how many?

13− 5=how many?
11− 6=how many?
13− 6=how many?
14− 7=how many?
16− 7=how many?
10− 8=how many?
12− 8=how many?
13− 9=how many?
17− 9=how many?

1. Subtract 375 from 796.

In this example, we place the subtrahend directly under the minuend, so that units stand under units, tens under tens, hundreds under hundreds. Now since each figure in the subtrahend is less than the corresponding figure in the minuend, we find no difficulty in making the subtraction, as in the annexed operation.

OPERATION.

Hundreds.	Tens.	Units.	
7	9	6	minuend.
3	7	5	subtrahend.
4	2	1	difference.

2. Subtract 495 from 867.

Having placed the subtrahend under the minuend, as in the last example, we commence with the unit figure 5 of the subtrahend, and subtract it from the corresponding figure of the minuend. Passing to the tens' figure of the subtrahend, which is 9, we see that it cannot be subtracted from the corresponding figure 6 of the minuend. By ART. **10**, we know that 10 in the place of tens is the same as one in the place of hundreds; we therefore increase the 6 of the minuend by 10, and diminish the 8 in the hundreds place by 1, the work then becomes as in the second operation. We now say 9 from 16 leaves 7, 4 from 7 leaves 3.

Instead of diminishing the 8 of the minuend by 1, we might have increased the 4 of the subtrahend by 1. The work will then be the same as in the third operation, which gives the same difference as before.

FIRST OPERATION.

Hundreds.	Tens.	Units.	
8	6	7	minuend.
4	9	5	subtrahend.
3	7	2	difference.

SECOND OPERATION.

Hundreds.	Tens.	Units.	
7	16	7	minuend.
4	9	5	subtrahend.
3	7	2	difference.

THIRD OPERATION.

Hundreds.	Tens.	Units.	
8	16	7	minuend.
5	9	5	subtrahend.
3	7	2	difference.

From the above operations we infer that,

When a figure in the subtrahend is greater than the corresponding figure of the minuend, we can subtract the subtrahend figure from the corresponding minuend figure, after it has been increased by 10. Before proceeding to subtract the next figure of the subtrahend, we must increase it by 1.

3. Subtract 3942 from 5678.

FIRST OPERATION.

Thousands.	Hundreds.	Tens.	Units.	
4	16	7	8	minuend.
3	9	4	2	subtrahend.
1	7	3	6	difference.

SECOND OPERATION.

Thousands.	Hundreds.	Tens.	Units.	
5	16	7	8	minuend.
4	9	4	2	subtrahend.
1	7	3	6	difference.

From what has been done we deduce this

RULE.

I. Place the subtrahend under the minuend, so that units may be directly under units, tens under tens, &c.

II. Then commencing at the right, subtract each figure of the subtrahend from the corresponding figure of the minuend; observing that when a figure of the subtrahend is greater than the corresponding figure of the minuend, to increase the minuend by 10 before subtracting, and then to carry 1 to the next figure of the subtrahend.

How do you place the numbers for subtraction? Where do you commence to subtract? Explain the method of subtracting when the figure in the subtrahend exceeds the corresponding figure of the minuend.

EXAMPLES.

4. From 34678 subtract 13787.

OPERATION.

34678
13787
20891 difference.

(5.)		(6.)	
789347		10345678937	
120305		902134124	
669042	difference.	9443544813	difference.

PROOF OF SUBTRACTION.

13. If the operation is rightly performed, the difference added to the subtrahend must equal the minuend.

	(7.)	(8.)	(9.)
	78543	612045	9345678201
	23056	137891	3279609167
Differences.	55487	474154	6066069034
Proofs.	78543	612045	9345678201

10. From seven million three hundred and sixty-five thousand two hundred and thirty-nine, take three hundred and forty-two thousand and thirteen. *Ans.* 7023226.

11. From one million and eleven, subtract thirteen. *Ans.* 999998.

2. From three hundred and sixty-five thousand, take three hundred and sixty-five. *Ans.* 364635.

13. America was discovered in 1492. How many years from that time to the year 1844? *Ans.* 352 years.

14. If a man receive 1134[illegible] dollars, and pay out of it 9203 dollars, how much will he have remaining? *Ans.* 2142 dollars.

15. In 1842 the Onondaga Salt Springs yielded 2291903 bushels of salt, and in 1826 they yielded 827505 bushels. How many more bushels were produced in 1842 than in 1826? *Ans.* 1464398 bushels.

16. In 1842 the United States shipped to England 205154 barrels of flour, to Scotland 3830 barrels. How many more barrels were sent to England than to Scotland? *Ans.* 201324 barrels.

17. Two men start together from the same place, and travel the same road; one goes 63 miles each dav. and the other goes 37 miles. How far apart will they the end of the first day? *Ans.* 26

18. George Washington was born in the year 1732; he died in the year 1799. To what age did he live?

Ans. 67 years.

19. At an election 12572 votes are taken, of which the successful candidate received 7391. How many votes did the other candidate receive? *Ans.* 5181 votes.

20. And what was the first one's majority?

Ans. 2210.

QUESTIONS INVOLVING ADDITION AND SUBTRACTION.

1. A lets B have 60 bushels of wheat worth 70 dollars, a fine horse worth 150 dollars, and 37 dollars' worth of butter. B in turn gives A his note for 110 dollars, and the rest in cash. What is the amount of cash?

Ans. 147 dollars.

2. A borrows of B, at one time, 375 dollars, at a second time he borrows 95 dollars, and at a third time, he borrows 413 dollars; he has paid him 319 dollars. How much does he still owe him? *Ans.* 564 dollars.

3. A person left a fortune of 10573 dollars to be divided between two sons and one daughter; the first son received 4309 dollars, the other son had 4987 dollars. How much did the daughter receive?

Ans. 1277 dollars.

4. Two persons are 375 miles apart, they travel toward each other; at the end of one day, one has travelled 93 miles, and the other 57 miles. How far apart are they?

Ans. 225 miles.

MULTIPLICATION OF SIMPLE NUMBERS.

14. Multiplication teaches to repeat one of two numbers as many times as there are units in the other.

The number to be repeated is called the *multiplicand*.

The number denoting how many times the multiplicand is to be repeated, is called the *multiplier*.

Both multiplicand and multiplier are called *factors*

The result obtained is called the *product*.

The symbol for multiplication is $\times$; this written between two numbers, indicates that they are to be multiplied together. Thus, 3×7 denotes that 3 is to be repeated 7 times, or, which is the same thing, 7 is to be repeated 3 times.

By the assistance of this symbol, we may form the following

MULTIPLICATION TABLE.*

$2\times 0=0$	$4\times 0=0$	$6\times 0=0$	$8\times 0=0$
$2\times 1=2$	$4\times 1=4$	$6\times 1=6$	$8\times 1=8$
$2\times 2=4$	$4\times 2=8$	$6\times 2=12$	$8\times 2=16$
$2\times 3=6$	$4\times 3=12$	$6\times 3=18$	$8\times 3=24$
$2\times 4=8$	$4\times 4=16$	$6\times 4=24$	$8\times 4=32$
$2\times 5=10$	$4\times 5=20$	$6\times 5=30$	$8\times 5=40$
$2\times 6=12$	$4\times 6=24$	$6\times 6=36$	$8\times 6=48$
$2\times 7=14$	$4\times 7=28$	$6\times 7=42$	$8\times 7=56$
$2\times 8=16$	$4\times 8=32$	$6\times 8=48$	$8\times 8=64$
$2\times 9=18$	$4\times 9=36$	$6\times 9=54$	$8\times 9=72$
$3\times 0=0$	$5\times 0=0$	$7\times 0=0$	$9\times 0=0$
$3\times 1=3$	$5\times 1=5$	$7\times 1=7$	$9\times 1=9$
$3\times 2=6$	$5\times 2=10$	$7\times 2=14$	$9\times 2=18$
$3\times 3=9$	$5\times 3=15$	$7\times 3=21$	$9\times 3=27$
$3\times 4=12$	$5\times 4=20$	$7\times 4=28$	$9\times 4=36$
$3\times 5=15$	$5\times 5=25$	$7\times 5=35$	$9\times 5=45$
$3\times 6=18$	$5\times 6=30$	$7\times 6=42$	$9\times 6=54$
$3\times 7=21$	$5\times 7=35$	$7\times 7=49$	$9\times 7=63$
$3\times 8=24$	$5\times 8=40$	$7\times 8=56$	$9\times 8=72$
$3\times 9=27$	$5\times 9=45$	$7\times 9=63$	$9\times 9=81$

This table should be committed to memory by the student.

* This table uses no factor consisting of more than one digit. I am aware that many tables of this kind are extended as far as 12 times 12, and others as far as 25 times 25, and even further; but I see no good reason why it should terminate at 12 times 12, any more than 13 times 13. I have therefore thought it better to limit it to 9 times 9, this being as far as it can extend by using but one digit as a factor. Still I have no objection to students committing to memory the products of as large factors as they may wish.

15. ***The student must also bear in mind that the multiplier and multiplicand may be interchanged without altering the product.*** Thus:

$$4\times8=8\times4=32 \qquad 4\times6=6\times4=24$$
$$9\times7=7\times9=63 \qquad 3\times5=5\times3=15.$$

What does multiplication teach? The number to be repeated is called what? The number denoting how many times the multiplicand is to be repeated is called what? What are the multiplicand and multiplier sometimes called? The result obtained is called what? What is the symbol for multiplication? Can the multiplier and multiplicand exchange places without altering the product?

When the multiplicand consists of more than one place of figures, and the multiplier has but one figure, we proceed as follows:

Multiply 697 by 3.

In this example, we first multiply 7, the figure in the place of units of the multiplicand, by the multiplier 3, and obtain 21 for the product, which we write down so that its right-hand figure is under the column of units. Next we multiply 9, the figure in the tens' place of the multiplicand, by the multiplier 3, and find 27 for the product, which we place so that the right-hand figure may be under the tens' column. Finally, we multiply 6, the figure of the hundreds' place of the multiplicand, by the multiplier 3, and find 18 for the product, which we write so that its right-hand figure may be under the column of hundreds.

OPERATION.

	Hundreds.	Tens.	Units.	
	6	9	7	multiplicand.
			3	multiplier.
		2	1	units.
	2	7		tens.
1	8			hundreds.
2	0	9	1	product.

We then add these partial products, and obtain 2091 for the total product.

By recalling to mind that ten in the place of units is equal to one in the place of tens, ten in the tens' place is equal to one in the hundreds' place, &c., we may perform the above multiplication as follows:

First, multiplying 7 of the multiplicand by 3 the multiplier, we get 21 units, which is the same as 2 tens and 1 unit; hence we write down the 1, and reserve the 2 to carry into the tens. Next, multiplying the 9 by 3, we find 27 tens, to which adding the 2, we have 29 tens, which is equal to 2 hundreds and 9 tens; we therefore write down the 9 and carry the 2 into the next product. Finally, multiplying 6 by 3, we find 18, to which adding the 2, we have 20, and as there is no farther multiplication, we write down the whole, and thus obtain the above result.

OPERATION.

697	multiplicand.
3	multiplier.
2091	product.

Again, let it be required to multiply 367 by 84, where the multiplier consists of more than one figure.

Multiplying first by the 4 which stands in the units' place, we find 1468 for the product. Next, multiplying by the 8 in the tens' place, we find 2936; since this product was obtained by multiplying by tens, its right-hand figure must be placed under the tens' column. Taking the sum of these partial products, we obtain the total product, 30828.

OPERATION.

367	multiplicand.
84	multiplier.
1468	
2936	
30828	product.

16. If we again take the first example, which is to multiply 697 by 3, we remark that since 697 is to be repeated 3 times, it may be done by writing it down 3 times, and then adding, thus:

$$\begin{array}{r} 697 \\ 697 \\ 697 \\ \hline 2091 \end{array}$$

And it is obvious that all questions of multiplication may be performed by addition.

Hence, multiplication is sometimes defined as being a concise way of performing several additions.

NOTE.—When a zero or 0 occurs in the multiplier, we may observe that its product must remain 0, since nothing repeated any number of times is still nothing.

PROOF OF MULTIPLICATION.

17. If we interchange the multiplier and multiplicand and then multiply, we shall obtain the same product if the work is right. (See ART. **15**.)

As in addition, these two results may be alike, and still the work may be wrong, since mistakes may occur in both operations. As good proof as any, is to carefully repeat the multiplication.

When 0 is multiplied by any number, what is the result? How is multiplication sometimes defined? How may multiplication be proved? Is this method infallible? Why not? What is as good proof as any other?

CASE I.

18. When the multiplier consists of only one figure.

From what has already been done, we deduce this

RULE.

Place the multiplying figure under the unit figure of the multiplicand, under which draw a horizontal line.

Then multiply each figure of the multiplicand by the multiplying figure, observing to carry one for every ten, as in addition.

When the multiplier consists of but one figure, how do you proceed? What rule do you observe in carrying?

EXAMPLES.

(1.)	(2.)	(3.)
1234	234156	612378
2	3	4
2468	702468	2449512

(4.)	(5.)	(6.)
897654	1003456	205670678
5	6	7
4488270	6020736	1439694746

(7.)	(8.)
6531023456	891030756078
8	9
52248187648	8019276804702

CASE II.

19. When the multiplier consists of more than one figure.

RULE.

I. Place the multiplier under the multiplicand, so that units may stand under units, tens under tens, hundreds under hundreds, &c.

II. Multiply successively by each figure of the multiplicand, as in Case I., observing to place the right-hand figure of each partial product directly under its multiplying figure.

III. Then add together these partial products, and the sum will be the total product sought.

When the multiplier consists of more than one figure, how do you write it? How do you then multiply? How do you add up?

EXAMPLES.

(1.)	(2.)	(3.)
23474	4567031	4005604
23	147	123
70422	31969217	12016812
46948	18268124	8011208
539902	4567031	4005604
	671353557	492689292

4. Multiply 12345 by 12. *Ans.* 148140.
5. Multiply 23456 by 11. *Ans.* 258016.
6. Multiply 34567 by 13. *Ans.* 449371.
7. Multiply 780056 by 21. *Ans.* 16381176.
8. Multiply 6503456 by 234. *Ans.* 1521808704.
9. Multiply 3471032 by 70056. *Ans.* 243166617792.
10. Multiply 1240578 by 302014. *Ans.* 374671924092.
11. Multiply 235678 by 753465. *Ans.* 177575124270.
12. Multiply 98610275 by 35789. *Ans.* 3529163131975.

CASE III.

20. When the multiplier, or multiplicand, or both, have one or more ciphers at the right.

We know from what has been said under ARTICLE 3, that multiplying by 10 is the same as annexing a cipher to the right, multiplying by 100 is the same as annexing two ciphers, &c.

Hence we deduce this

RULE.

Multiply by the significant figures, as in Case II., and to the product annex as many ciphers as there are in both multiplier and multiplicand.

When there are ciphers to the right of the multiplier, or multiplicand, or both, how do you proceed?

EXAMPLES.

1. Multiply 365 by 10. *Ans.* 3650.
2. Multiply 12040 by 100. *Ans.* 1204000.
3. Multiply 204500 by 3000. *Ans.* 613500000.
4. Multiply 7003000 by 240000. *Ans.* 1680720000000.
5. Multiply 307210000 by 3780000. *Ans.* 1161253800000000.

CASE IV.

21. When the multiplier is a composite number.

A composite number is one which may be produced by multiplying two or more numbers together. Thus: 35 is a composite number, which may be produced by multiplying 5 and 7 together.

The 5 and 7 are called the *factors* or component parts of 35.

The factors of 12, are 3 and 4, or 2 and 6.

Suppose we wish to multiply 48 by 35.

If we first multiply 48 by 5, we find 240 for the product; if now we multiply this product by 7, we obtain 1680, which is evidently the same as 35 times 48.

Hence we infer this

RULE.

Multiply successively by each factor; the last product will be the one sought.

EXAMPLES.

1. Multiply 365 by 28.

The factors of 28 are 4 and 7. Hence we have this

OPERATION.

```
  365
    4 one of the component parts.
 ----
 1460
    7 the other component part.
-----
10220 Ans.
```

2. Multiply 374 by $24 = 4 \times 6 = 3 \times 8 = 2 \times 12 = 2 \times 3 \times 4$.

FIRST OPERATION.

```
      374
        4 1st component part.
     ----
     1496
        6 2d component part.
     ----
Ans. 8976
```

SECOND OPERATION.

```
      374
        3 1st component part.
     ----
     1122
        8 2d component part
     ----
Ans. 8976
```

THIRD OPERATION.

```
      374
        2 1st component part.
     ----
      748
       12 2d, component part.
     ----
     1496
     748
     ----
Ans. 8976
```

FOURTH OPERATION.

```
      374
        2 1st component part.
     ----
      748
        3 2d component part.
     ----
     2244
        4 3d component part.
     ----
Ans. 8976
```

From the above examples, we see that it makes no difference how we resolve the multiplier into factors, provided we multiply in succession by all the factors.

What is a composite number? What are the component parts? How do you proceed when the multiplier is a composite number? Does it make any difference which component part we first multiply by?

3. Multiply 345678 by $36 = 6 \times 6 = 4 \times 9 = 3 \times 12 = 3 \times 3 \times 4$. *Ans.* 12444408.

4. Multiply 1002456 by $72=8\times9=2\times3\times3\times4=2\times2\times2\times3\times3$. *Ans.* 72176832.

5. Multiply 7540102 by $84=7\times12=3\times4\times7=2\times2\times3\times7$. *Ans.* 633368568.

EXERCISES IN MULTIPLICATION.

1. Suppose I buy 15 loads of brick, each load containing 1250 brick, how many brick have I?
Ans. 18750 brick.

2. In an orchard there are 107 apple-trees, each producing 19 bushels of apples. How many bushels does the whole orchard yield? *Ans.* 2033 bushels.

3. If a person travel 17 days at the rate of 37 miles each day, how many miles will he travel in all?
Ans. 629 miles.

4. If a person buy 175 barrels of salt, each weighing 304 pounds, how many pounds in all will he have?
Ans. 53200 pounds.

5. Suppose I purchase the following bill of merchandise:
3 Firkins of butter, each 15 dollars.
7 Hogsheads of molasses, each 23 dollars.
12 Bags of coffee, each 11 dollars.
5 Boxes of raisins, each 2 dollars.
3 Boxes of lemons, each 5 dollars.
How many dollars must I give for the whole?
Ans. 363 dollars.

6. How many dollars will the following bill of goods amount to?
52 Yards of black broadcloth, at 4 dollars per yard.
40 Yards of Brussels carpeting, at 2 dollars per yard.
2 Sofas, each 56 dollars.
9 Mahogany chairs, each 5 dollars.
5 French bedsteads, each 7 dollars. *Ans.* 480 dollars.

DIVISION OF SIMPLE NUMBERS.

22. DIVISION teaches the method of finding how many times one number is contained in another.

The number to be divided is called the *dividend.*

The number by which we divide is called the *divisor.*

The number of times which the dividend contains the divisor, is called the *quotient.*

Besides these three parts there is sometimes a *remainder*, which is of the same name as the dividend, since it is a part of it.

The sign usually employed to indicate division is ÷ Thus, 12÷3, denotes that 12 is to be divided by 3.

By using this sign we may form the following

DIVISION TABLE.

2÷2=1	3÷3=1	4÷4=1	5÷5=1
4÷2=2	6÷3=2	8÷4=2	10÷5=2
6÷2=3	9÷3=3	12÷4=3	15÷5=3
8÷2=4	12÷3=4	16÷4=4	20÷5=4
10÷2=5	15÷3=5	20÷4=5	25÷5=5
12÷2=6	18÷3=6	24÷4=6	30÷5=6
14÷2=7	21÷3=7	28÷4=7	35÷5=7
16÷2=8	24÷3=8	32÷4=8	40÷5=8
18÷2=9	27÷3=9	36÷4=9	45÷5=9
6÷6=1	7÷7=1	8÷8=1	9÷9=1
12÷6=2	14÷7=2	16÷8=2	18÷9=2
18÷6=3	21÷7=3	24÷8=3	27÷9=3
24÷6=4	28÷7=4	32÷8=4	36÷9=4
30÷6=5	35÷7=5	40÷8=5	45÷9=5
36÷6=6	42÷7=6	48÷8=6	54÷9=6
42÷6=7	49÷7=7	56÷8=7	63÷9=7
48÷6=8	56÷7=8	64÷8=8	72÷9=8
54÷6=9	63÷7=9	72÷8=9	81÷9=9

23. Division may also be represented by placing the divisor under the dividend, with a short horizontal line between them; thus, $\frac{10}{2}$ denotes that 10 is to be divided by 2.

In the same way we have

$\frac{12}{2}=12\div2$; $\frac{13}{3}=13\div3$; $\frac{17}{5}=17\div5$; $\frac{53}{7}=53\div7$.

This method is employed, when in division there is a remainder, to express accurately the value of such remainder.

What does division teach? What is the number to be divided called? What is the number by which we divide called? What is the number of times which the dividend contains the divisor called? There is sometimes another part, what is it? Of what name is the remainder? What is the symbol of division? By what other method is division denoted?

When the divisor consists of only one figure, we proceed as follows:

Divide 973 by 7.

Having placed the divisor at the left of the dividend, keeping them separate by means of a curved line, we draw a straight horizontal line underneath.

OPERATION.

7)973
  139 quotient.

We then, first, see how many times 7 is contained in 9, which is 1 time and 2 remainder; we place the 1 directly underneath, and conceive the 2 to be prefixed to the next figure of the dividend, making 27. Next, we see how many times 7 is contained in 27, which is 3 times and 6 remainder; we place the 3 for the next figure of the quotient, and conceive the 6 to be prefixed to the next figure of the dividend, making 63. Finally, we see how many times 7 is contained in 63, which we find to be exactly 9 times; we therefore write 9 for the last figure of the quotient.

From this operation we find that 7 is contained 139 times in 973. Hence, 139 repeated 7 times ought to equal 973, which we find to be the case.

24. Suppose we wish to know how many times 8 is contained in 32. We might proceed as follows: since 32 is greater than 8, we know that 8 is contained in it, at least once, therefore, subtracting 8 from 32, we find 24 for a remainder. Again, we know that 8 is contained at least once in 24; therefore, subtracting 8 from 24, we have 16, from which, subtracting 8, we have left 8; finally,

from 8 subtracting 8, we have no remainder. Hence, we perceive that 8 has been subtracted 4 times from 32, that is, 8 is contained just four times in 32. It is obvious that by continued subtractions any sum in division may be performed.

For this reason division is said to be a concise way of performing several subtractions.

CASE I.

25. Short division, or when the divisor consists of only one figure.

From the above operation we infer the following

RULE.

I. Place the dividing figure at the left of the dividend, keeping them separated by a curved line, and draw a straight horizontal line underneath them.

II. Seek how many times the dividing figure is contained in the left-hand figure or figures of the dividend, which must be placed directly beneath, for the first figure of the quotient.

III. If there is no remainder, divide the next figure of the dividend for the next figure of the quotient. But when there is a remainder, conceive it to be prefixed to the next succeeding figure of the dividend, before making the following division. If a figure of the dividend, which is required to be divided, is less than the dividing figure, we must write 0 in the quotient, and consider this figure as a remainder.

Division is said to be a concise way of performing what? What is short division? Repeat the rule.

EXAMPLES.

1. Divide 2345675 by 8.

OPERATION.

divisor 8)2345675 dividend.
quotient 293209 with 3 remainder

26 When there is a remainder, we may place it over

the divisor, with a short horizontal line between them, thus indicating that this remainder is still to be divided by the divisor, agreeably to ART. **23**.

2. Divide 12456789 by 4.	*Ans.* $3114197\frac{1}{4}$.
3. Divide 78900346 by 7.	*Ans.* 11271478.
4. Divide 131305678 by 6.	*Ans.* $21884279\frac{4}{6}$.
5 Divide 357020348 by 3.	*Ans.* $119006782\frac{2}{3}$.

CASE II.

27. Long division, or when the divisor consists of more than one figure.

RULE.

I. Place the divisor at the left of the dividend, keeping them separate by a curved line.

II. Seek how many times the divisor is contained in the fewest figures of the dividend; set the figure expressing the number of times to the right of the dividend, keeping them separate by means of a curved line, for the first figure of the quotient.

III. Multiply the divisor by this quotient figure, and subtract the product from the first figures of the dividend, and to the remainder annex the next figure of the dividend; then find how many times the divisor is contained in this new number, and write the figure in the quotient.

IV. Again, multiply the divisor by this last quotient figure, and subtract the product from the last number which contained the divisor, and to the remainder annex the next figure of the dividend. Thus continue the operation until all the figures of the dividend have been brought down.

NOTE 1. After having brought down a new figure, if the whole is then less than the divisor, it will contain it 0 times; we must therefore write 0 in the quotient, and bring down another figure.

NOTE 2. If any of the partial products are greater than the number which was supposed to contain the divisor, the quotient figure must be taken smaller.

NOTE 3. If we obtain a remainder larger than the divisor, our quotient figure must be taken larger.

EXAMPLES.

1. Divide 4703598 by 354.

OPERATION.

```
Divisor. Dividend. Quotient.
      354)4703598(13287
          354      first product.
          ---
          1163
          1062     second product.
          ----
           1015
            708    third product.
           ----
            3079
            2832   fourth product.
            ----
             2478
             2478  fifth product.
             ----
```

If now we multiply the divisor by the quotient, we have this

OPERATION.

```
      354
    13287
    -----
     2478   first product.
    2832    second product.
   708      third product.
  1062      fourth product.
  354       fifth product.
  -------
  4703598
```

Here we discover that the products obtained by this multiplication, are the same as those obtained in the operation of division, only they occur in a reverse order. In the operation of division, each succeeding product is placed one figure farther toward the right, while in the operation of multiplication, each succeeding product is placed one figure farther toward the left. Hence the sum of the products in the case of division, must be the same as the sum in the case of multiplication. In the operation of division, by the above rule, these products

are successively subtracted from the corresponding parts of the dividend, until the whole is exhausted. Now we have just shown by the operation of multiplication, that the sum of these products, taken in the order in which they stand, is equal to the dividend. Therefore the above rule for *long division* must be correct.

PROOF.

From what has been said, we also infer that this method of long division proves itself as we proceed with the work, since we have only to add the successive products, and the remainder, if any, to obtain the dividend.

What is long division? How do you place the numbers? Repeat the rule. If, after having brought down a new figure, the result is less than the divisor, how do you proceed? When the partial product is greater than the number which was supposed to contain the divisor, how do you do? When the remainder is greater than the divisor, how do you proceed? Explain the method of proof.

2. Divide 175678 by 223.

OPERATION.

```
223)175678(787
    1561  first product.
    ----
     1957
     1784  second product.
     ----
      1738
      1561  third product.
      ----
       177  remainder.
```

If we take the sum of the successive products and the remainder, adding them as they now stand in the above work, we shall obtain 175678; which, agreeing with the dividend, proves the accuracy of the division. This method of proving division, is perhaps, as simple and brief as any method which can be devised.

The common method of proving division, and one which is applicable to short division as well as long division, is to multiply the divisor and quotient together, and to add in the remainder, if any.

3. Divide 7892343 by 139. *Ans.* $56779\frac{62}{139}$.
4. Divide 177575124270 by 753465. *Ans.* 235678.
5. Divide 34789205 by 64534. *Ans.* $539\frac{5379}{64534}$.
6. Divide 123456789 by 789. *Ans.* $156472\frac{381}{789}$.
7. Divide 5763447 by 678509. *Ans.* $8\frac{335375}{678509}$.
8. Divide 1521808704 by 6503456. *Ans.* 234.
9. Divide 243166625648 by 3471032.
Ans. 70056 with 7856 remainder.
10. Divide 166168212890625 by 12890625.*
Ans. 12890625.
11. Divide 11963109376 by 109376. *Ans.* 109376.

CASE III.

28. When the divisor is a composite number.

In this case we evidently have the following

RULE.

Divide successively by each of the factors of the divisor. It makes no difference which factor we first use.

To obtain the true remainder, we must observe the following

RULE.

Multiply the last remainder by the preceding divisor, and add in the preceding remainder; multiply this sum by the next preceding divisor, and add in the next preceding remainder; so continue this reverse process until you have multiplied by all the divisors.

How do you proceed when the divisor is a composite number? Does it make any difference which factor we first divide by? When there are several remainders, explain how the true remainder is obtained.

EXAMPLES.

1. Divide 839 by 120.

In this case we will resolve 120 into the three factors $4\times5\times6=120$. Now proceeding agreeably to the rule, we have this

* This question and the next are worthy of notice, since the terminal figures of the dividend, divisor, and quotient, are the same.

OPERATION.

4)839
5)209 3=first remainder.
6)41 4=second remainder.
6 5=third remainder.

Now, to obtain the true remainder, we have this

OPERATION.

last remainder. preceding divisor. preceding remainder. next preceding divisor. next preceding remainder. true remainder.

$5 \times 5 + 4 = 29$ again, $29 \times 4 + 3 = 119$

Had there been more than three factors, the method of operation would have been equally as simple, but a little more lengthy.

2. Divide 8217 by 35.

The factors of 35 are 5 and 7. Hence, we have this

OPERATION.

5)8217
7)1643 2=first remainder.
234 5=last remainder.
$5 \times 5 + 2 = 27$=true remainder.

3. Divide 33678 by $15 = 3 \times 5$.

OPERATION.

3)33678
5)11226 no remainder.
2245 1=last remainder.
$1 \times 3 = 3$ true remainder.

4. Divide 9591 by $72=8\times 9$.

Ans. 133 with 15 remainder.

5. Divide 10859 by $49=7\times 7$.

Ans. 221 with 30 remainder.

CASE IV.

29. When the divisor ends with one or more ciphers.

We have seen under ART. **3**, that a number is multiplied by 10 by annexing a cipher; it is multiplied by 100 by annexing two ciphers; by 1000 by annexing three ciphers, &c. Conversely, a number is divided by 10 by cutting off one figure from the right; it is divided by 100 by cutting off two figures from the right, &c. Hence we have this

RULE.

Cut off from the right-hand of the dividend, as many figures as there are ciphers in the divisor, then divide what remains by the significant figures of the divisor after the ciphers are omitted. To the remainder bring down the figures cut off from the dividend. This will give the true remainder.

How do you proceed when there are ciphers at the right of the divisor?

EXAMPLES.

1. Divide 4567894 by 3700.

OPERATION.

```
37|00)45678|94(1234 quotient.
      37
       86
       74
       127
       111
        168
        148
        2094 remainder.
```

If we regard the divisor 3700 as a composite number whose factors are $100 \times 37 = 3700$, this example will properly fall under the last Case. According to which the 94 cut off from the right of the dividend is to be considered the *first remainder*, and the 20 is the *last remainder*. Hence the true remainder is $20 \times 100 + 94 = 2094$, from which we discover the correctness of the above rule.

2. Divide 7123545 by 421000.

Ans. 16 with 387545 remainder.

3. Divide 1212121212 by 42000.

Ans. 28860 and 1212 remainder.

4. Divide 123456789 by 12300.

Ans. 10037 and 1689 remainder

EXERCISES IN DIVISION.

1. In one year there are 365 days, and in one week there are 7 days. How many weeks in one year?

Ans. 52 weeks and 1 day.

2. Nine square feet make one square yard. How many square yards are there in 495 square feet?

Ans. 55 square yards.

3. Three men are to share equally in the sum of 1236 dollars. How many dollars will each have?

Ans. 412 dollars.

4. Divide 1245 acres of land equally between five brothers. *Ans.* Each has 249 acres.

5. It is about 95000000 miles from here to the sun. Now admitting that it requires 8 minutes for light to pass from the sun to the earth, how many miles does it pass in one minute? *Ans.* 11875000 miles.

6. Allowing 22 brick to be sufficient to make one cubic foot of masonry, how many cubic feet are there in a work which requires 100000 brick?

Ans. 4545 cubic feet and 10 brick remaining.

7. The circumference of the earth is about 24900 miles. How long would it require for a person to travel around it, if he could pass uninterruptedly at the rate of 200 miles per day? *Ans.* $124\frac{1}{2}$ days.

QUESTIONS EXERCISING THE FOUR GROUND RULES.

1. A person owes to one man 375 dollars, to another he owes 708 dollars, to a third man he owes 911 dollars. How much does he owe to the three men? *Ans.* 1994 dollars.

2. A farmer has sheep in five fields; in the first, he has 917; in the second, 249; in the third, 413; in the fourth, 1000; and in the fifth, he has 197. How many sheep has he in the five fields? *Ans.* 2776 sheep.

3. A person owes to one man 302 dollars, to another man he owes 707 dollars, and has owing to him 2000 dollars. How much will remain after paying his debts?

Ans. 991 dollars.

4. A farmer receives for his wheat 103 dollars, for his corn 60 dollars, for his butter 511 dollars, for his cheese 1212 dollars, for his pork 601 dollars. He pays toward a new farm 1000 dollars, for a new wagon 50 dollars, for hired help on his farm 290 dollars, for repairing house 173 dollars. How much money has he remaining?

Ans. 974 dollars.

5. A person wills 1200 dollars to his wife, 300 dollars for charitable purposes, and what remains is to be equally divided among 6 children. Allowing his property to amount to 8562 dollars, how much would each child have?

Ans. 1177 dollars.

6. A man gave 13558 dollars for a farm, he then sold 73 acres at 75 dollars per acre, the remainder stood him in at 59 dollars per acre. How many acres did he purchase?

Ans. 210 acres.

7. Four boys divide 336 apples as follows: the first takes one sixth of the whole; the second takes one fourth of what was left; the third takes one half of what was then left; the fourth has the remainder. What number of apples did each boy have?

Ans. { The first had 56.
The second had 70.
The third had 10
The fourth had 1[illegible]

8. An estate of 8100 dollars was divided among 9 cl dren in the following way: the first had 100 dollars

one tenth of the remainder; after this the second had 200 dollars and one tenth of the residue; again, the third had 300 dollars and one tenth of the remainder, and so on; each succeeding child had 100 dollars more than the one immediately preceding, and then one tenth of what still remained. What was the share of each?

Ans. { They shared equal, each had 900 dollars.

9. A and B each owe C: A owes 1472 dollars, which is less than what B owes him, and yet the difference between A's and B's debts is 719 dollars. How much does B owe C? *Ans.* 2191 dollars.

10. Admitting the earth to move 68000 miles per hour, how far will it move in one day; and how far in a year of 365 days?

Ans. { 1632000 miles in one day.
595680000 miles in one year.

FRACTIONS.

30. A fraction is a part of a unit.

Several methods are used to express fractions or parts of units, which give rise to several distinct kinds of fractions. Those usually employed in arithmetic are

VULGAR OR COMMON FRACTIONS,

AND

DECIMAL FRACTIONS.

VULGAR FRACTIONS.

31. Vulgar fractions consist of two distinct parts or terms, the one written above the other, with a straight horizontal line between them, as in division, ART. **23**. The number above the line is called the *numerator*. The number below the line is called the *denominator*. The denom-

inator shows how many parts the unit is divided into; and the numerator shows how many parts are used.

Thus $\frac{5}{8}$ is a vulgar fraction, whose numerator is 5 and denominator 8: it is read *five eighths*

A vulgar fraction may be considered a concise method of expressing division (ART. **23**), where the numerator corresponds to the dividend, and the denominator to the divisor. Thus $\frac{5}{8}$ is the same as 5 divided by 8, and it may therefore be read *one eighth of five*, or, as above, *five eighths of one*. In the same way $\frac{1}{9}$ indicates that 1 is divided into 9 equal parts: it is read *one ninth of one*. After the same manner,

$\frac{3}{7}$ is read *one seventh of three*, or *three sevenths of one*.
$\frac{4}{5}$ is read *one fifth of four*, or *four fifths of one*.
$\frac{6}{11}$ is read *one eleventh of six*, or *six elevenths of one*.
$\frac{8}{9}$ is read *one ninth of eight*, or *eight ninths of one*.
&c. &c. &c.

The fraction	$\frac{5}{7}$	denotes that	5	is to be divided by	7.
"	$\frac{13}{4}$	"	13	"	4.
"	$\frac{17}{8}$	"	17	"	8.
"	$\frac{3}{12}$	"	3	"	12.
"	$\frac{1}{2}$	"	1	"	2.
"	$\frac{1}{3}$	"	1	"	3.
"	$\frac{1}{4}$	"	1	"	4.
"	$\frac{2}{5}$	"	2	"	5.
	&c.		&c.		&c.

When the numerator is equal to the denominator, the value of the fraction is a unit.

When the numerator is less than the denominator, the value is less than a unit, and the expression is called a *proper fraction*.

When the numerator is greater than the denominator, the value is greater than a unit, and the expression is called an *improper fraction*.

Thus, each of the expressions $\frac{3}{3}$, $\frac{5}{5}$, $\frac{10}{10}$, $\frac{13}{13}$, &c., is equal to a unit.

Each of the expressions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{6}{7}$, $\frac{1}{10}$, &c., is a proper fraction.

Each of the expressions $\frac{2}{1}$, $\frac{7}{3}$, $\frac{8}{5}$, $\frac{12}{11}$, $\frac{9}{2}$, $\frac{17}{13}$, &c., is an improper fraction.

When a whole number and fraction are connected, the expression is called a *mixed number*. Thus, $4\frac{1}{2}$, $3\frac{1}{7}$, $5\frac{1}{17}$, $2\frac{20}{27}$, &c., are mixed numbers.

When several fractions are connected by the word *of*, the expression is called a *compound fraction*. The expressions $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{9}$, $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{8}{5}$ of $\frac{11}{10}$, $\frac{3}{7}$ of $\frac{3}{8}$ of $\frac{3}{9}$ of $\frac{5}{6}$, &c., are compound fractions.

Any number may be made to assume the form of an improper fraction, by writing under it a unit for the denominator. Thus, 2, 3, 4, 5, 7, &c., are the same as $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$, $\frac{5}{1}$, $\frac{7}{1}$, &c.

Fractions sometimes occur, in which the numerator, or denominator, or both, are themselves fractional; such expressions are called *complex fractions*.

Thus, $\frac{3\frac{1}{2}}{4}$, $\frac{4}{7\frac{1}{3}}$, $\frac{2\frac{1}{5}}{3\frac{1}{7}}$, $\frac{10\frac{1}{9}}{9\frac{1}{10}}$, &c., are complex fractions.

A fraction is said to be inverted when the numerator and denominator exchange places. Thus: the fractions $\frac{2}{3}$, $\frac{5}{4}$, $\frac{7}{8}$, $\frac{6}{10}$, $\frac{4}{9}$, $\frac{3}{7}$, when inverted, become $\frac{3}{2}$, $\frac{4}{5}$, $\frac{8}{7}$, $\frac{10}{6}$, $\frac{9}{4}$, $\frac{7}{3}$.

What is a fraction? What two methods are usually employed to express fractions? What is a vulgar fraction? Which is the numerator of a vulgar fraction? Which the denominator? What does the denominator show? What does the numerator show? In the vulgar fraction five eighths, which is the numerator, and which the denominator? How is it read? What may a vulgar fraction be considered a concise way of expressing? In a vulgar fraction, which part corresponds to the dividend, and which to the divisor? What is the value of the fraction, when the numerator is equal to the denominator? When is the value less than a unit? What is the fraction then called? When is the value greater than a unit? What is the fraction then called? Give examples of proper fractions. Give examples of improper fractions. When a whole number and fraction are connected, what is the expression called? Give examples. When several fractions are connected by the word *of*, what kind of a fraction is it then called? Give examples. When the numerator, or denominator, or both, are already fractional, what are they called? Give examples. When is a fraction said to be inverted? Give examples.

Having defined vulgar fractions, we shall now treat of decimal fractions, which are more nearly allied to whole

numbers; and shall hereafter resume the subject of vulgar fractions.

DECIMAL FRACTIONS.

32. A decimal fraction is that particular form of a vulgar fraction, whose denominator consists of a unit, followed by one or more ciphers.

Thus: $\frac{1}{10}$, $\frac{3}{10}$, $\frac{4}{100}$, $\frac{37}{100}$, $\frac{8}{100}$, $\frac{3}{1000}$, $\frac{47}{10000}$, &c., are decimal fractions.

In practice the denominators of decimal fractions are not written, but always understood.

The above decimal fractions are usually written as follows: 0.1, 0.3, 0.04, 0.37, 0.08, 0.003, 0.0047, &c.

The period, or decimal point, serves to separate the decimals from the whole numbers.

The first figure on the right of the period, or *decimal point*, is said to be in the place of tenths; the second figure is said to be in the place of hundredths; the third figure in the place of thousandths, and so on, decreasing from the left toward the right, in a ten-fold ratio, the same as in whole numbers. The following table will exhibit this more clearly:

NUMERATION TABLE OF WHOLE NUMBERS AND DECIMALS.

&c., &c.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Hundred Millionths.	Billionths.	Ten Billionths.	&c., &c.
	3	3	3	3	3	3	3	3	3	3	3	.	3	3	3	3	3	3	3	3	3	3	

Ascending. ☜ ☞ Descending.

This table is in accordance with the French method of numbering (ART. 7), where each period of three figures changes its denominate value.

Since decimals, like whole numbers, decrease from the left toward the right in a ten-fold ratio, they may be con-

nected together by means of the decimal point, and then operated upon by precisely the same rules as for whole numbers, provided we are careful to keep the decimal point always in the right place.

Annexing a cipher to a decimal does not change its value. Thus, 0.3=0.30=0.300=&c. But prefixing a cipher, is the same as removing the decimal figures one place farther to the right, and therefore, each cipher thus prefixed reduces the value in a ten-fold ratio.

Thus, 0.3 is ten times 0.03, or a hundred times 0.003.

0.2	is read	two tenths.
0.25	"	twenty-five hundredths.
0.365	"	three hundred and sixty-five thousandths.
0.105	"	one hundred and five thousandths.
0.03	"	three hundredths.
0.047	"	forty-seven thousandths.
0.1234	"	one thousand two hundred and thirty-four ten thousandths.
4.3	"	four and three tenths.
37.3	"	thirty-seven and three tenths.
365.03	"	three hundred and sixty-five and three hundredths.
&c.,	&c.,	&c.

6.4	is the same as	$6\frac{4}{10}$
36.5	"	$36\frac{5}{10}$
36.05	"	$36\frac{5}{100}$
0.7	"	$\frac{7}{10}$
0.37	"	$\frac{37}{100}$
0.123	"	$\frac{123}{1000}$
0.2345	"	$\frac{2345}{10000}$
0.0101	"	$\frac{101}{10000}$
0.00012	"	$\frac{12}{100000}$
0.40056	"	$\frac{40056}{100000}$
40.0005	"	$40\frac{5}{10000}$
101.0101	"	$101\frac{101}{10000}$.

A number composed of a whole number and a decimal part, is called a *mixed number*.

What is a decimal fraction? Of what form is the denominator? Give examples of decimal fractions. In practice which part is not written, but understood? What purpose does the decimal point serve? What place is the first figure on the right of the decimal point said to occupy? What place does the second figure occupy? What place does the third figure occupy? In what ratio do the values decrease in passing to the right? Is the above table in accordance with the French, or English method of notation? Does annexing a cipher to a decimal alter its value? What effect is produced by prefixing a cipher? A number which is composed of a whole number and decimal is called what?

ADDITION OF DECIMAL FRACTIONS.

33. Since decimals, like whole numbers, increase from the right towards the left, they may be treated by the same rules as for whole numbers, provided we are careful to keep the decimal point in the right place. Hence we have this

RULE.

Place the numbers so that the decimal points shall be directly over each other, and then add as in whole numbers.

How do you place the numbers to be added? How is the work performed?

EXAMPLES.

(1.)

	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.
	3	7	4	1.	1	2	5	0
	1	4	1	2.	1	3	4	6
	2	3	1	0.	2	0	0	5
	4	1	0	0.	1	0	1	6
	3	4	5	6.	4	3	1	2
1	5	0	1	9.	9	9	2	9

(2.)

	Units.	Tenths.	Hundredths.	Thousandths.	Tens of Thousandths.	Hund. Thousandths.
	1.	2	3	4	5	6
	2.	3	4	5	6	7
	3.	4	5	6	7	8
	4.	5	6	7	8	9
	5.	6	7	8	0	0
1	7.	2	8	2	9	0

(3.)	(4.)	(5.)
4123.245	0.43478	11.111
1.12	1.35001	210.001
37.004	1.1	8.8
0.205	33.333	9.808
4161.574	36.21779	239.720

6. What is the sum of 0.123, 0.012, 0.675, 0.0045? *Ans.* 0.8145.

7. What is the sum of 0.14145, 0.23235, 0.34345, 0.45455? *Ans.* 1.17180.

8. Find the sum of 1.0012, 23.1003, 101.31407, 10.101578. *Ans.* 135.517148.

9. Find the sum of 234.12, 23.412, 2.3412, 0.23412. *Ans.* 2[illegible]0732.

10. What is the sum of 111.111, 12.1212, 13.1313, 14.1414? *Ans.* 150.5049.

SUBTRACTION OF DECIMAL FRACTIONS.

34. There is no difference between the subtraction of decimals and that of whole numbers, provided we are careful to keep the decimal points directly under each other, so that like denominations may stand under each other. Hence this

RULE.

Place the less number under the greater, so that the decimal points shall be, the one directly under the other; then subtract, as in whole numbers.

How do you place the numbers in subtraction? Then how do you proceed

EXAMPLES.

(1.)	(2.)	(3.)
345.345	1245.3478	3456.12347846
54.123	340.0122	479.100345
291.222	905.3356	2977.02313346

4. From 1023.4, subtract 99.9. *Ans.* 923.5.
5. From 0.4785, subtract 0.13047. *Ans.* 0.34803.
6. From 0.11234, subtract 0.00675. *Ans.* 0.10559.

MULTIPLICATION OF DECIMAL FRACTIONS.

35. It is obvious that tenths multiplied by tenths must give hundredths. Tenths by hundredths must give thousandths. Hundredths by hundredths must give ten thousandths, and so on. So that when decimals are multiplied together, there will be as many places of decimals in the product as there are in both the factors. We therefore have this

RULE.

Multiply as in whole numbers, and give as many decimal places in the product, as there are in both the factors taken together. When there are not as many places in the product, prefix ciphers.

How do you multiply decimals? How many decimal places must there be in the product? When the whole number of figures in the product is not as great, how do you proceed?

EXAMPLES.

1. Multiply 0.125 by 0.37.

OPERATION.

$$\begin{array}{r} 0.125 \\ \underline{0.37} \\ 875 \\ \underline{375} \\ \underline{0.04625} \end{array}$$

In this example, the multiplicand has 3 decimal places, and the multiplier has 2; therefore, by the rule, the product must have 5 places, and since the product consists of but 4 figures, we prefix one cipher before making the decimal point.

2. Multiply 0.561 by 0.786. *Ans.* 0.440946.
3. Multiply 3.012 by 4.027. *Ans.* 12.129324.
4. Multiply 47.051 by 37.039. *Ans.* 1742.721989.
5. Multiply 33.33 by 66.66. *Ans.* 2221.7778.
6. Multiply 125.125 by 5.5. *Ans.* 688.1875.

36. A decimal number may be multiplied by 10, 100, 1000, &c., by removing the decimal point as many places to the right as there are ciphers in the multiplier; and if there are not so many figures, make up the deficiency by annexing ciphers.

$$\text{Thus, 12.12 multiplied by} \left\{\begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \\ 1000000 \end{array}\right\} = \left\{\begin{array}{r} 121.2 \\ 1212. \\ 12120. \\ 121200. \\ 1212000. \\ 12120000. \end{array}\right.$$

How may a decimal number be multiplied by 10, 100, 1000, &c.? When there are not as many decimal figures in the multiplicand as there are ciphers in the multiplier, how do you proceed?

DIVISION OF DECIMAL FRACTIONS.

37. In multiplication of decimals, we know that the number of decimal places in the product is equal to the sum of those in both the factors. Now since the product divided by one of the factors must produce the other factor or quotient, it follows, that in division the decimal places of the dividend must be equal to the number of places in the divisor and quotient taken together. Hence, the number of decimal places in the quotient must equal the excess of those in the dividend above those in the divisor. Therefore, we obviously have this

RULE.

Divide as in whole numbers, and give as many decimal places in the quotient as those in the dividend exceed those in the divisor; if there are not as many, supply the deficiency by prefixing ciphers.

How do you divide one decimal by another? How many decimal places must the quotient have? If the whole number of figures in the quotient is not as great as the number of decimals required, how do you proceed?

EXAMPLES.

1. Divide 0.123428 by 11.8.

OPERATION.

```
11.8)0.123428(0.01046.
     118
     ---
      542
      472
      ---
       708
       708
       ---
```

In this example, the dividend contains 6 decimal places, and the divisor but 1; therefore, by the rule the quotient ought to contain 5, but as there are but 4 figures in the quotient, we make up the deficiency by prefixing a cipher before making the decimal point.

2. Divide 3.810688 by 1.12. *Ans.* 3.4024.
3. Divide 0.109896 by 0.241. *Ans.* 0.456.
4. Divide 1.12264556 by 1.0012. *Ans.* 1.1213.
5. Divide 0.01764144 by 0.0018. *Ans.* 9.8008.

38. When there are not as many decimal places in the dividend as in the divisor, we may by ART. **32** annex as many ciphers to the dividend as we please, if we do not change the place of the decimal point. When the number of decimal places are the same in both dividend and divisor, the quotient will be a whole number.

When there are not as many decimal places in the dividend as in the divisor, how do you proceed? When the number of decimal places in the dividend is the same as in the divisor, what will the quotient be?

6. Divide 244.431 by 1.2345.

In this example, before performing the division, we annex a cipher to the dividend so that it may have as many decimal places as the divisor has; we then perform this

OPERATION.

```
1.2345)244.4310(198 whole number.
       12345
       -----
       120981
       111105
       ------
         98760
         98760
         -----
```

7. Divide 122.418 by 3.4005. *Ans.* 36.
8. Divide 0.7 by 0.07. *Ans.* 10.
9. Divide 0.25 by 0.0005. *Ans.* 500.
10. Divide 0.125 by 0.000005. *Ans.* 25000.

39. When there is still a remainder, and we wish a more accurate quotient, we may continue to annex ciphers and to divide as far as we please, observing the rule for placing the decimal point.

11. Divide 20 by 0.003.

OPERATION.

```
By short division.   0.003)20.000
                          6666.6666 to any extent.
```

12. Divide 37.4 by 4.5.

OPERATION.

```
4.5)37.4(8.31111+
    360
    ---
    140
    135
    ---
      50
      45
      --
       50
       45
       --
        50
        45
        --
         50
         45
         --
          5
```

Where, in the quotient, we have written the sign + it is to indicate that the quotient is still larger than is written.

It frequently happens, as in this example, that the work will never terminate.

When there is still a remainder, how may we proceed to obtain a still more accurate value for the quotient? What does the sign + at the right of a quotient indicate?

13. Divide 7.85 by 3.43. *Ans.* 2.2886+.
14. Divide 0.478 by 0.58. *Ans.* 0.824+.
15. Divide 0.9009 by 0.4051. *Ans.* 2.223+.

40. We may, obviously, divide any decimal by 10, 100, 1000, &c., by removing the decimal point as many places to the left as there are ciphers in the divisor; when there are not so many figures to the left of the decimal point, we may prefix ciphers.

$$\text{Thus, 12.12 divided by} \left\{\begin{matrix} 10 \\ 100 \\ 1000 \\ 10000 \\ 100000 \\ 1000000 \end{matrix}\right\} = \left\{\begin{matrix} 1.212 \\ 0.1212 \\ 0.01212 \\ 0.001212 \\ 0.0001212 \\ 0.00001212 \end{matrix}\right.$$

How may we divide a decimal by 10, 100, 1000, &c.? When in the decimal number there are not as many figures on the left of the decimal point as there are ciphers in the divisor, how do you proceed?

FEDERAL MONEY.

41. This is the currency of the United States.

Its denominations, or names, are Eagles, Dollars, Dimes, Cents, and Mills.

$$\left.\begin{matrix} \text{Eagles,} \\ \text{Half eagles,} \\ \text{Quarter eagles,} \end{matrix}\right\} \text{are coined from gold.}$$

$$\left.\begin{matrix} \text{Dollars,} \\ \text{Half dollars,} \\ \text{Quarter dollars,} \\ \text{Dimes,} \\ \text{Half dimes,} \end{matrix}\right\} \text{are coined from silver.}$$

Cents, }
Half cents, } are coined from copper.

The mill is never coined.

42. The gold for coinage is not pure, but consists of $\frac{22}{24}$ of pure gold, $\frac{1}{24}$ of silver, and $\frac{1}{24}$ of copper; or, as usually expressed, 22 carats of gold, 1 of silver, and 1 of copper.

A carat being $\frac{1}{24}$ part of the whole.

The standard for silver is 1489 of pure silver, to 179 of pure copper; which, in carats, is $21\frac{59}{139}$ of silver, and $2\frac{80}{139}$ of copper.

The copper coins are of pure copper.

TABLE OF FEDERAL MONEY.

10 mills *m*	make	1 cent,		*ct.*
10 cents	"	1 dime,		*d.*
10 dimes	"	1 dollar,		$
10 dollars	"	1 eagle,		*E.*

m.	*ct.*			
10=	1	*d.*		
100=	10=	1	$	
1000=	100=	10=	1	*E.*
10000=	1000=	100=	10=	1.

Where is Federal money used? What are its denominations? Which are coined from gold? Which from silver? Which from copper? Which one is never coined? What metals are mixed with gold for coining? In gold coins, what is the ratio of the copper and silver to the gold? What is a carat? What is the standard for silver coins? What is the ratio when estimated in carats? Is the copper for copper coins also alloyed? Repeat the table of Federal Money.

43. Since the different denominations succeed each other in a ten-fold ratio, as in whole numbers and decimals, it is plain that the preceding rules for decimals are applicable to this currency. Federal money ought never to be treated as denominate numbers, since it is by far the simplest and best way to consider its denominations the same as decimals. To make this more clear, we will give the following table of Federal money:

TABLE.

&c. &c.	Thousands of dollars.	Hundreds of dollars.	Tens of dollars, or eagles.	Dollars.		Tenths of a dollar, or dimes.	Hundredths of a dollar, or cents.	Thousandths of a dollar, or mills.	
	4	4	4	4	.	4	4	4	=$4444, 44 cents, 4 mills.
		4	4	4	.	4	4	4	=$444, 44 cents, 4 mills.
			4	4	.	4	4	4	=$44, 44 cents, 4 mills.
				4	.	4	4	4	=$4, 44 cents, 4 mills.
				0	.	4	4	4	=44 cents, 4 mills.
				0	.	0	4	4	=4 cents, 4 mills.
				0	.	0	0	4	=4 mills.

It is customary in accounts to use only dollars, cents, and mills, so that eagles are expressed in dollars; and dimes in cents.

In what ratio do the different denominations of Federal Money decrease? Are the rules for decimals applicable to this currency? Should Federal Money be treated as denominate numbers? In accounts, which denominations only are used? How then are eagles expressed? How are dimes expressed?

Thus: 5 eagles and six dollars is the same as 56 dollars.
4 dimes and 5 cents is the same as 45 cents.
3 dimes 3 cents and 3 mills is the same as 333 mills.
2 dimes and 2 mills is the same as 202 mills.
1 dollar is the same as 100 cents, which is 1000 mills.
2 dollars is the same as 200 cents, which is 2000 mills.
5 dollars is the same as 500 cents, which is 5000 mills.
7 dollars is the same as 700 cents, which is 7000 mills.

56 dollars is the same as 5600 cents, which is 56000 mills.

365 dollars is the same as 36500 cents, which is 365000 mills.

3456 dollars is the same as 345600 cents, which is 3456000 mills.

&c. &c.

From this we see that dollars are changed into cents by annexing two ciphers, cents are changed into mills by annexing one cipher, and dollars into mills by annexing three ciphers.

How are dollars changed into cents? How are cents changed into mills? How are dollars changed into mills?

EXAMPLES.

1. How many cents in $6? *Ans.* 600.
2. How many mills in 13 cents? *Ans.* 130.
3. How many mills in $4 and 45 cents? *Ans.* 4450.
4. How many mills in 75 cents and 1 mill? *Ans.* 751.
5. How many cents in $9 and 13 cents? *Ans.* 913.
6 How many mills in $5 and 55 cents and 5 mills. *Ans.* 5555.

44. If we cut off one from the right of mills, which is dividing by 10 (Art. **40**), they will be changed into cents; if from the right of cents we cut off two, that is, divide by 100 they will be changed into dollars; and if we cut off three from the right of mills, that is, divide by 1000, they will be changed into dollars.

How may mills be changed to cents? How may cents be changed to dollars? How may mills be changed to dollars?

EXAMPLES.

1. How many dollars in 113 cents? *Ans.* $1.13.
2. How many dollars in 12345 mills? *Ans.* $12.345.
3. How many dollars in 1004 mills? *Ans.* $1.004.
4. How many cents in 45678 mills? *Ans.* 4567.8 cents.
5. How many dollars in 2456405 mills? *Ans.* $2456.405.

TABLE

OF SOME FRACTIONAL PARTS OF A DOLLAR FREQUENTLY USED.

5	cents $=\frac{1}{20}$ of a dollar.
$6\frac{1}{4}$	cents $=\frac{1}{16}$ of a dollar.
10	cents $=\frac{1}{10}$ of a dollar.
$12\frac{1}{2}$	cents $=\frac{1}{8}$ of a dollar.
$16\frac{2}{3}$	cents $=\frac{1}{6}$ of a dollar.
20	cents $=\frac{1}{5}$ of a dollar.
25	cents $=\frac{1}{4}$ of a dollar.
$33\frac{1}{3}$	cents $=\frac{1}{3}$ of a dollar.
50	cents $=\frac{1}{2}$ of a dollar.
100	cents $=1$ dollar.

QUESTIONS WROUGHT BY DECIMALS.

1. Bought 4 loads of wood, the first contained 0.97 cords. the second contained 1.03 cords, the third contained 0.945 cords, the fourth contained 1.005 cords. What did the four loads measure? *Ans.* 3.95 cords

2. In the month of May the amount of rain was 3.15 inches, in June it was 4.05 inches, in July it was 2.97 inches, and in August it was 3.03 inches. How much rain fell during these four months? *Ans.* 13.2 inches.

3. During three successive days the mean range of the barometer was 29.04 inches, 29.51 inches, and 29.73 inches. What is the sum of these heights?
Ans. 88.28 inches.

4. Bought a box of raisins for $1.75, one bushel of apples for $0.375, one cheese for $3.175, one barrel of sugar for $15.50. What did the whole amount to?
Ans. $20.80.

5. A farmer receives $15.375 for a cow, $75 for a fine horse, $3.125 for some potatoes, $5.55 for some poultry. How much did he receive in all? *Ans.* $99.05.

6. A person bought some velvet for $3.333, some broadcloth for $18.75, some silk for 12.50, some cotton

cloth \$5.405, one shawl \$12.25, some carpeting \$30.05. What did the whole amount to? *Ans.* \$82.288.

7. A person borrowed \$213.375, of which he has paid \$107.18. How much does he still owe? *Ans.* \$106.195.

8. Bought a cow for \$13.25, paid \$6.875. How much remains unpaid? *Ans.* \$6.375.

9. What will 185 pounds of coffee cost, at \$0.138 per pound? *Ans.* \$25.53.

10. Bought 8.375 cords of wood, at \$2.50 per cord What did it cost? *Ans.* \$20.9375.

11. What will 121.5 gallons of molasses come to, at 41 cents per gallon? *Ans.* \$49.815.

12. The length of the Erie Canal is 364 miles, and it cost \$7143790. What was the average expense of one mile? *Ans.* \$19625.796+.

13. Crooked Lake Canal is 8 miles long, and cost \$156777. How much is this per mile?

Ans. \$19597.125.

14. In 1842, the whole number of children taught in the district schools of the State of New-York was 598901; the whole amount disbursed for common schools was \$1155419.90. How much was that per scholar?

Ans. \$1.929+.

15. The salary of the President of the United States is \$25000. How much is that each day?

Ans. \$68.493+.

16. In one rod there are 16.5 feet. How many rods in 3573 feet? *Ans.* 216.5454+ rods.

17. Bought a farm of 137 acres for \$5324. How much was that per acre? *Ans.* \$38.861+.

45. To find the value of articles estimated by the 100, or 1000.

What is the value of 9425 brick, at \$3.25 per 1000?

Had the price been \$3.25 for each brick, we should multiply the number of brick by this price, thus:

OPERATION.

```
     9425
     3.25
    47125
   18850
  28275
$30631.25
```

This value of $30631.25 is evidently 1000 times too much; therefore, to obtain the true value, we must divide it by 1000, which is done, (ART. 40,) by removing the decimal point three places to the left; it will then become $30.63125. Had they been $3.25 per 100, then instead of removing the decimal point three places to the left, we should have removed it two places. Hence we have this

RULE.

Multiply the number of articles by the price, by the 100, *or* 1000, *and from the product cut off two of the right-hand figures when the articles are estimated by the* 100, *and three when they are estimated by the* 1000.

EXAMPLES.

1. What is the value of 1300 feet hemlock boards, at $5.50 per 1000?

OPERATION.

```
   1300
   5.50
  65000
 65
$7.15000
```

In this example, we cut off two for decimals, and three because the articles are estimated by the 1000, so that the whole number cut off is five.

2. What is the value of 675 feet clear pine stuff, at $25 per 1000? *Ans.* $16.875.

3. What is the value of 11035 feet of timber, at $2.25 per 100? *Ans.* $248.2875.

4. What is the value of 90422 brick, at $3.75 per 1000? *Ans.* $339.0825.

5. What must be paid for laying 875 brick, at $3.25 per 1000? *Ans.* $2.84375.

DENOMINATE NUMBERS.

46. Simple numbers are expressions for a certain number of units, without regard to the particular value of the unit. Thus, 37 is the same as 37 times one, abstractly considered; it does not mean 37 times a yard, pound, foot, or any other particular unit.

A denominate number always expresses the particular *kind* of unit. Thus, 8 yards is a denominate number whose unit is one yard; 8 pounds is a denominate number whose unit is one pound.

Several numbers of different denominations are frequently grouped together, as 6 feet 3 inches.

All our different kinds of weights and measures are denominate numbers. It is much to be regretted that we are obliged to employ such a variety of different measures, when the same end would be accomplished by one measure for weight, and one for each of the three geometrical magnitudes, lengths, surfaces, and solids.

The French have adopted such a system of weights and measures, graduated on the decimal scale of notation.

What is a simple number? What is a compound number? What kind of numbers are all our different weights and measures? How many kinds of weights and measures have the French adopted?

The following are some of the most important tables of weights and measures at present employed in this country.

ENGLISH MONEY.

47. The denominations of English money are Farthings, Pence, Shillings, and Pounds.

The pound is, however, never coined, but there is a gold coin equivalent in value to one pound, called a sovereign.

TABLE.*

4 farthings	*far.*	make	1 penny,	*d.*
12 pence		"	1 shilling,	*s.*
20 shillings		"	1 pound,	£

far.	*d.*		
4=	1	*s.*	
48=	12=	1	£
960=	240=	20=	1

NOTE.—Farthings are sometimes expressed in fractions of a penny, as follows: 1 farthing $=\frac{1}{4}$ *d.*, 2 farthings $=\frac{1}{2}$ *d.*, 3 farthings $=\frac{3}{4}$ *d.*

What are the denominations of English money? Which denomination is never coined? What gold coin is equivalent in value to one pound? Repeat the Table. How are farthings sometimes expressed?

TROY WEIGHT.†

48. Coins, precious metals, jewels, and liquors, are weighed by this weight.

---

* The full weight and value of English gold and silver coin is as follows:

	Name of Coin.	*Value.* £	*s.*	*d.*	*Weight.* *pwt.*	*gr*
Gold,	A guinea,	1	1	0	5	$9\frac{1}{2}$
	Half guinea,	0	10	6	2	$16\frac{3}{4}$
	Quarter guinea,	0	5	3	1	$8\frac{1}{4}$
	Sovereign,	1	0	0	5	$3\frac{3}{11}$
	Half sovereign,	0	10	0	2	$13\frac{7}{11}$
Silver.	A crown,	0	5	0	18	$4\frac{4}{11}$
	Half crown,	0	2	6	9	$2\frac{2}{11}$
	Shilling,	0	1	0	3	$15\frac{3}{11}$
	Sixpence,	0	0	6	1	$19\frac{7}{11}$

† The original of all weights used in England was a grain, or corn of wheat, gathered out of the middle of the ear, and being well dried, 32 of them were to make one pennyweight, 20 pennyweights one ounce, and 12 ounces one pound. But in later times, it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in common use.

TABLE.

24 grains *gr.*	make	1	pennyweight,	*pwt.*
20 pennyweights	"	1	ounce,	*oz.*
12 ounces	"	1	pound,	*lb.*

gr.	*pwt.*		
24 =	1 *oz.*		
480 =	20 =	1 *lb.*	
5760 =	240 =	12 =	1

What substances are weighed by Troy weight? Repeat the Table.

APOTHECARIES' WEIGHT.

49. This weight, as its name would imply, is used in weighing medicines. Its pound and ounce is the same as in Troy Weight.

TABLE.

20 grains *gr.*	make	1	scruple,	℈.
3 scruples	"	1	dram,	ʒ.
8 drams	"	1	ounce,	℥.
12 ounces	"	1	pound,	℔.

gr.	℈			
20 =	1 ʒ			
60 =	3 =	1 ℥		
480 =	24 =	8 =	1 ℔	
5760 =	288 =	96 =	12 =	1

For what purpose is Apothecaries' Weight used? Does its pound and ounce differ from Troy Weight?

AVOIRDUPOIS WEIGHT.*

50. By this weight are weighed all things of a coarse or drossy nature, as bread, butter, cheese, flesh, grocery wares, and some liquids; all metals, except gold and silver.

It appears from the table that 112 pounds make one

---

	oz.	*pwt.*	*gr.*	*gr.*	
* 1 *lb.* Avoirdupois =	14	11	16	= 7000	Troy.
1 *oz.* " =	0	18	$5\frac{1}{2}$ =	$437\frac{1}{2}$	Troy.
1 *dr.* " =	0	1	$3\frac{11}{32}$ =	$27\frac{11}{32}$	Troy.

hundred weight. But in most cases, at the present time, 100 pounds is reckoned instead of 112.

TABLE.

16 drams	*dr.*	make	1 ounce,	*oz.*
16 ounces		"	1 pound,	*lb.*
28 pounds		"	1 quarter,	*qr.*
4 quarters		"	1 hundred weight,	*cwt.*
20 hundred weight		"	1 ton,	*T.*

dr.	*oz.*				
16=	1	*lb.*			
256=	16=	1	*qr.*		
7168=	448=	28=	1	*cwt.*	
28672=	1792=	112=	4=	1	*T.*
573440=	35840=	2240=	80=	20=	1

What substances are weighed by Avoirdupois Weight? Repeat the Table. By this weight how many pounds make one hundred weight?

LONG MEASURE.

51. TABLE.

3	barleycorns *bar.*	make	1 inch,	*in.*
12	inches	"	1 foot,	*ft.*
3	feet	"	1 yard,	*yd.*
$5\frac{1}{2}$	yards	"	1 rod, perch, or pole,	*rd.*
40	rods	"	1 furlong,	*fur.*
8	furlongs,	"	1 mile,	*mi.*
3	miles,	"	1 league,	*L.*
*$69\frac{1}{6}$	miles, nearly,	"	1 degree,	*deg.* or °.

in.	*ft.*				
12=	1	*yd.*			
36=	3 =	1	*rd.*		
198=	$16\frac{1}{2}$=	$5\frac{1}{2}$=	1	*fur.*	
7920=	660 =	220 =	40=	1	*mi.*
63360=	5280 =	1760 =	320=	8=	1

Repeat the Table of Long Measure.

* The latest measurements give the equatorial diameter of the earth equal to 7925.648 miles. Consequently, its circumference is 24899 miles, which, divided by 360, gives the length of a degree $69\frac{1}{6}$ miles, nearly.

CLOTH MEASURE.

52.

TABLE.

$2\frac{1}{4}$ inches *in.*	make	1	nail,	*na.*
4 nails	"	1	quarter of a yard,	*qr.*
3 quarters	"	1	Ell Flemish,	*E. Fl.*
4 quarters	"	1	yard,	*yd.*
4 *qr.* $1\frac{1}{2}$ *in.*	"	1	Ell Scotch,	*E. S.*
5 quarters	"	1	Ell English,	*E. E.*
6 quarters	"	1	Ell French,	*E. Fr.*

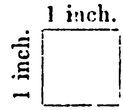
Repeat the Table of Cloth Measure.

SQUARE MEASURE.

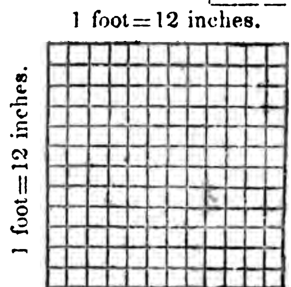
53. This measure is used for estimating artificers' work, such as boards, glass, pavements, plastering, flooring, painting, and any other kind of work where length and breadth only are concerned. It is always employed for measuring land, and for this reason it is sometimes called *Land Measure.*

A square is a figure having four equal sides, and all its angles *right,* that is, the sides are perpendicular to each other.

If the length of one of the sides is one inch, as in the adjoining figure, then it is called a *square inch*



If the length of one of the sides is one foot, or 12 inches, it is called a *square foot,* which by the adjacent figure we see is composed of $12 \times 12 = 144$ *square inches.*



In a similar manner, if we had a square, each of whose sides was 3 feet, then it would contain $3 \times 3 = 9$ sq. *feet,* which is called one *square yard,* since 3 feet = 1 yard.

TABLE.

144	square inches	*Sq. in.* make	1	square foot,	*Sq. ft.*
9	square feet	"	1	square yard,	*Sq. yd.*
$30\frac{1}{4}$	square yards	"	1	square pole,	*P.*
40	square poles	"	1	square rood,	*R.*
4	square roods	"	1	acre,	*A.*
640	acres	"	1	square mile,	*M.*

Sq. in.	*Sq. ft.*				
144=	1	*Sq. yd.*			
1296=	9 =	1	*P.*		
39204=	$272\frac{1}{4}$=	$30\frac{1}{4}$=	1	*R.*	
1568160=	10890 =	1210 =	40=	1	*A.*
6272640=	43560 =	4840 =	160=	4=	1

In measuring land, Gunter's chain is used; its length is 4 rods, or 66 feet. It is divided into 100 links.

$7\frac{92}{100}$ inches	make	1	link,	*l*
100 links, or 4 rods, or 66 feet,	"	1	chain,	*c*
80 chains	"	1	mile,	*m.*
10000 square links	"	1	square chain	
10 square chains	"	1	acre,	*A.*

What use is made of Square Measure? When employed in measuring land how is it called? What is a square? When a square is one inch on each side how is it called? When it is one foot or 12 inches on each side how is it called? When it is one yard on each side how is it called? Repeat the Table of Square Measure. In Land Measure with what are the sides of the field usually measured? How long is this chain? Repeat the Table of Land Measure.

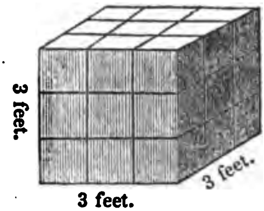
SOLID, OR CUBIC MEASURE.

54. This is used in measuring all bodies where we have regard to length, breadth, and thickness, such as earth, stone, timber, &c.

A Cube is a solid bounded by six equal squares, resembling a common tea-chest.

If the sides of a cube are each one inch long, it is

called a *cubic inch.* If each side is one foot, it is called a *cubic foot.* If a side is one rod, it is called a *cubic rod.*



In the adjoining figure we have endeavored to represent a cube, each side of which is 3 feet, or one yard, and consequently it is one *cubic* yard.

The top, which is equal to the base, contains $3\times3=9$ square feet; hence, if this was only one foot in height, it would contain 9 cubic feet; but as it is 3 feet in height, it must therefore contain 3 times $9=27$ cubic feet. Hence, one cubic yard is equivalent to $3\times3\times3=27$ cubic feet.

In the same way one cubic foot is equivalent to $12\times12\times12=1728$ cubic inches.

TABLE.

1728 solid inches	*S. in.*	make	1 solid foot,	*S. ft.*
27 solid feet		"	1 solid yard,	*S. yd.*
*40 feet of round timber or 50 feet of hewn timber		"	1 ton,	*Ton.*
128 solid feet		"	1 cord of wood,	*C.*

A pile of wood 4 feet wide, 4 feet high, and 8 feet long, will make 1 cord. One foot in length of such a pile is sometimes called *a cord foot.* It contains 16 solid feet; consequently 8 cord feet make one cord.

For what is Solid Measure used? What is a Cube? In a cubic yard how many cubic feet? In a cubic foot how many cubic inches? How many cubic feet of round timber make a ton? How many of hewn timber? How many cubic feet make a cord of wood? Explain what is meant by a cord foot.

WINE MEASURE.

55. By this are measured all liquids except beer.

* A ton of round timber is so much as, when hewed, shall make 40 cubic feet.

TABLE.

4	gills	*gi.*	make	1 pint,	*pt.*
2	pints		"	1 quart,	*qt.*
4	quarts		"	1 gallon,	*gal.*
31½	gallons		"	1 barrel,	*bar.*
63	gallons		"	1 hogshead,	*hhd.*
2	hogsheads		"	1 pipe,	*pi.*
2	pipes		"	1 tun,	*tun*

gi	*pt.*						
4=	1	*qt.*					
8=	2=	1	*gal.*				
32=	8=	4=	1	*bar.*			
1008=	252=	126=	31½=	1	*hhd.*		
2016=	504=	252=	63 =	2=	1	*pi.*	
4032=	1008=	504=	126 =	4=	2=	1	*tun.*
8064=	2016=	1008=	252 =	8=	4=	2=	1

The wine gallon contains 231 cubic or solid inches.

What liquids are measured by Wine Measure? Repeat the Table. How many cubic inches in the wine gallon?

ALE, OR BEER MEASURE.

56.

TABLE.

2	pints	*pt.*	make	1 quart,	*qt.*
4	quarts		"	1 gallon,	*gal.*
36	gallons		"	1 barrel,	*bar.*
1½	barrels		"	1 hogshead,	*hhd.*

pt.	*qt.*			
2=	1	*gal.*		
8=	4=	1	*bar.*	
288=	144=	36=	1	*hhd.*
432=	216=	54=	1½=	1

The beer gallon contains 282 cubic or solid inches.

What is measured by Beer Measure? Repeat the Table. How many cubic inches in the beer gallon?

DRY MEASURE.

57. By this are measured all dry wares, as grain, seeds, roots, fruits, salt, coal, sand, oysters, &c.

TABLE.

2 pints	*pt.*	make	1 quart,	*qt.*
8 quarts		"	1 peck,	*pk.*
4 pecks		"	1 bushel,	*bu.*
*36 bushels		"	1 chaldron,	*ch.*

pt.	*qt.*			
2=	1	*pk.*		
16=	8=	1	*bu.*	
64=	32=	4=	1	*ch.*
2304=	1152=	144=	36=	1

By the English statute the dry gallon must contain $268\frac{4}{5}$ cubic or solid inches. The corn or Winchester bushel must contain $2150\frac{2}{5}$ cubic or solid inches. This measure is of a cylindric form, 8 inches deep and $18\frac{1}{2}$ inches in diameter.

By an act of Parliament which took effect the 1st of January, 1826, the imperial gallon of 277.274 cubic inches was adopted as the only gallon.

According to the Revised Statutes of the state of New York, a cubic foot of distilled water, when estimated under prescribed circumstances, is to consist of $62\frac{1}{2}$ pounds, or 1000 ounces avoirdupois weight. Eight pounds of such water is to constitute the gallon for liquid measure, and ten pounds is to make the gallon for dry measure.

What articles are measured by Dry Measure? Repeat the Table. How many cubic inches in the dry gallon, according to the English statute? How many cubic inches in a bushel? What is the form and dimensions of the Winchester bushel measure? How many cubic inches in the English imperial gallon? How many pounds of water constitute the dry gallon, according to the Revised Statute of New York? How many pounds make the liquid gallon?

* In the United States 32 bushels = 1 chaldron.

TIME.

58.

TABLE.

60 second	*sec.*	make	1 minute,	*min.*
60 minutes		"	1 hour,	*hr.*
24 hours		"	1 day,	*da.*
7 days		"	1 week,	*wk.*
4 weeks		"	1 month,	*mo.*
13 mo. 1 da. 6 hr. or 365 da. 6 hr.		"	1 Julian year,	*yr.*

sec.	*min.*				
60=	1	*hr.*			
3600=	60=	1	*da.*		
86400=	1440=	24=	1	*wk.*	
604800=	10080=	168=	7 =	1	*yr.*
31557600=	525960=	8766=	$365\frac{1}{4}$=	$52\frac{5}{28}$=	1

The true length of the solar year, to the nearest secon is 365 da. 5 hr. 48 m. 48 sec.

The civil year is also divided into 12 calendar month as follows:

			Days.
1	month	January, - - -	31
2	"	February, - - -	28 or 29
3	"	March, - - -	31
4	"	April, - - -	30
5	"	May, - - - -	31
6	"	June, - - -	30
7	"	July, - - - -	31
8	"	August, - - -	31
9	"	September, - - -	30
10	"	October, - - -	31
11	"	November, - - -	30
12	"	December, - -	31
			365 or 366

If the year exceeded 365 days by 6 hours exactly, th once in four years these hours would amount to anoth

day. Hence, once in four years an additional day is given to the month of February; and such years are called Bissextile or Leap Years. But, since this excess is not quite 6 hours, this rule of adding one day to February every fourth year is interrupted, and the centennial years, which are not divisible by 400, are regarded as common years.* Hence, any year, except a centurial year, which is divisible by 4, is a Leap year, or consists of 366 days.

Centennial years which will divide by 400 are regarded as Leap years; all others are considered as common years.

1796, 1804, 1808, 1812, 1816, 1820, 1824, 1828, 1832, 1836, 1840, were all Leap years. 1800 not being divisible by 400, was a common year of 365 days; the same may be said of 1900; but the year 2000, being divisible by 400, will be a Leap year.

Thirty days hath September,
April, June, and November—
All the rest have thirty-one,
Excepting February alone,
Which has but twenty-eight in fine,
Till Leap year gives it twenty-nine.

Repeat the Table for Time. What is the length of the solar year to the nearest second? Into how many calendar months is the civil year divided? Repeat their names and the number of days belonging to each. How often in general is an additional day added to February? What are such years styled? Is the rule of counting every fourth year Leap year correct? Are centurial years which are not divisible by 400 Leap years? Was 1800 a Leap year? Mention the next preceding and next following Leap year to 1800.

CIRCULAR MEASURE, OR MOTION.

59. By this is estimated Latitude and Longitude, and the motion of the heavenly bodies which appear to move in

* There is still a further modification which takes place at the end of every 1000 years, which it is unnecessary to explain in his place.

circles. Every circle, whether great or small, is divided into 360 degrees.

TABLE.

60 seconds	″	make	1 minute,	′
60 minutes		"	1 degree,	°
30 degrees		"	1 sign,	s.
12 signs or 360°		"	1 circle,	c.

″	′	°	s.	c.
60=	1			
3600=	60=	1		
108000=	1800=	30=	1	
1296000=	21600=	360=	12=	1

What use is made of Circular Motion? Into how many degrees are all circles supposed to be divided? Repeat the Table.

60. Measures, &c., not included in the foregoing tables.

6 points	make	1 line	} used in measuring length of clock pendulum rods.
12 lines	"	1 inch	
4 inches	"	1 hand	used in measuring the height of horses.
6 feet	"	1 fathom	used in measuring depths at sea.

12	individual things	make	1 dozen.
12	dozen	"	1 gross.
12	gross	"	1 great gross.
20	individual things	"	1 score.
24	sheets of paper	"	1 quire.
20	quires	"	1 ream.
112	pounds	"	1 quintal of fish.
200	"	"	1 barrel of pork, or beef.
196	"	"	1 barrel of flour.

Repeat the above tables.

BOOKS.

61. A sheet folded into two leaves is called a folio.
" folded into four leaves is called a quarto, or 4to.
" folded into eight leaves is called an octavo, or 8vo.
" folded into twelve leaves is called a duodecimo, or 12mo.
" folded into eighteen leaves is called an 18mo.

When a sheet is folded into two leaves what is it called? How called when folded into four leaves? How when folded into eight leaves? How when folded into twelve leaves? How when folded into eighteen leaves?

REDUCTION.

62. Reduction is the changing of numbers from one name or denomination to another, without altering their value.

When the denominations are to be reduced from a higher denomination to a lower, it is called *Reduction Descending*; but when they are to be reduced from a lower to a higher denomination, it is called *Reduction Ascending*.

REDUCTION DESCENDING.

Let it be required to reduce £7 5*s.* 10*d.* 3 *far.*, to farthings.

OPERATION.

	7	pounds.
	20	shillings in one pound.
	140	product in shillings.
add	5	shillings.
	145	
	12	pence in one shilling.
	290	
	145	
	1740	product in pence.
add	10	pence.
	1750	
	4	farthings in one penny.
	7000	product in farthings.
add	3	farthings.
	7003	number of farthings sought.

From the above operation, we readily deduce this general

RULE.

Multiply the number in the highest denomination by as many of the next lower as make one in that higher; to this product add the number, if any, belonging to this lower denomination; we shall thus obtain an equivalent value in the next lower denomination.

II. Proceed in a similar way for all the successive denominations, until we reach the last; which last result will be the number sought.

What is reduction? When is it called descending? And when ascending? Repeat the rule for Reduction descending.

REDUCTION ASCENDING.

63. Let it be required to reverse the last example, that is, to find the number of pounds, shillings, pence, and farthings, in 7003 farthings.

We must obviously perform a reverse operation to that performed under reduction descending.

OPERATION.

```
    far.
  4)7003
    ----
    1750d. 3 far. remainder.

      d.    s.
  12)1750(145
     12
     --
      55
      48
      --
       70
       60
       --
       10d. remainder.

                 s.
            2|0)14|5
            --------
                £7  5s. remainder.
```

Collecting results, we have 7003 farthings, equivalent to £7 5*s.* 10*d.* 3 *far.*

EXPLANATION.

First, we divide the number of farthings, 7003, by 4, because 4 farthings make one penny; the quotient is 1750 pence, and 3 farthings remaining.

Secondly, we divide the number of pence, 1750, by 12, because 12 pence make one shilling; the work being performed by long division, we get for the quotient 145 shillings, and 10 pence remaining.

Thirdly, we divide the number of shillings, 145, by 20, because 20 shillings make one pound; cutting off the cipher from the right of 20, and the right-hand figure from the dividend, (ART. 29,) we perform the work by short division, and obtain the quotient, 7 pounds, and 5 shillings remaining.

We may, therefore, deduce this general

RULE.

I. Divide the given number by as many of that denomination as make one of the next higher; write down the quotient and remainder, if any.

II. Divide the quotient by as many of this denomination as make one of the next higher; write this new quotient and the remainder as before.

III. Proceed in this way through all the denominations to the highest, and the quotient last found, together with the several remainders, if any, will give the value sought.

Repeat the Rule for Reduction Ascending.

EXAMPLES.

1. In £47 5*s*. 2*d*. 1 *far*. how many farthings?

OPERATION.

```
£47   5s.  2d.  1 far.
  20
 945  shillings.
  12
1892
945
11342 pence.
    4
45369 farthings.
```

2. In 118567 farthings, how many pounds, shillings, pence, and farthings?

OPERATION.

far.
4)118567
29641 3 farthings.
d. s.
12)29641(2470
24
56
48
84
84
1 penny.
s.
2|0)247|0
£123 10 shillings.

Hence, 118567 farthings are equal to £123 10*s*. 1*d*. 3 *far*.

3. Reduce £75 to shillings. *Ans*. 1500*s*.

4. Reduce 19*s*. 6*d*. to pence. *Ans*. 234*d*.

5. Reduce 15*s*. 3*d*. 2 *far*. to farthings. *Ans*. 734 *far*.

6. In 48926 grains, Troy Weight, how many pounds, ounces, pennyweights, and grains? *Ans*. 8*lb*. 5*oz*. 18*pwt*. 14*gr*.

7. In 3605 pennyweights, how many pounds, ounces, and pennyweights? *Ans*. 15*lb*. 0*oz*. 5*pwt*.

8. In 1000 ounces, Troy Weight, how many pounds and ounces? *Ans*. 83*lb*. 4*oz*.

9. In 4*lb*. 6*oz*. 13*pwt*. 5*gr*. how many grains? *Ans*. 26237*gr*.

10. In 100*lb*. 1 *gr*. how many grains? *Ans*. 576001*gr*.

11. In 4 ℔ 5 ℥ 1 ʒ how many drams? *Ans*. 425 ʒ.

12. In 1000 grains, Apothecaries' Weight, how many ounces, drams, scruples, and grains? *Ans*. 2 ℥ 0ʒ 2℈.

13. In 11521 grains, Apothecaries' Weight, how many pounds? *Ans.* 2℔ 0℥ 0ʒ 0℈ 1*gr.*

14. In 873450 drams, Avoirdupois Weight, how many tons? *Ans.* 1*T.* 10*cwt.* 1*qr.* 23*lb.* 14*oz.* 10*dr.*

15. Reduce 5*cwt.* 21*lb.* 4*oz.* to ounces. *Ans.* 9300 ounces.

16. Reduce 1*T.* 1*cwt.* 1*dr.* to drams. *Ans.* 602113 drams.

17. Reduce 856702 drams to tons. *Ans.* 1*T.* 9*cwt.* 3*qr.* 14*lb.* 7*oz.* 14*dr.*

18. In 4355 inches, how many yards? *Ans.* 120*yds.* 2*ft.* 11*in.*

19. In 248 miles, how many inches? *Ans.* 15713280 inches.

20. How many inches in 360 degrees of $69\frac{1}{6}$ miles to each degree, which is the circumference of the earth, nearly? *Ans.* 1577664000.

21. In 12121212 barleycorns, how many miles? *Ans.* 63*mi.* 6*fur.* 6*rd.* 0*yd.* 1*ft.* 4*in.*

22. Reduce 12 Ells French to nails. *Ans.* 288 nails.

23. Reduce 11 Ells English, 3 quarters, to quarters. *Ans.* 58 quarters.

24. Reduce 10 Ells Flemish, 3 quarters, 1 nail, to nails. *Ans.* 133 nails.

25. Reduce 4 yards to quarters. *Ans.* 16 quarters.

26. In 1000 nails, how many yards? *Ans.* 62*yds.* 2*qr.*

27. How many inches in 6 yards, 3 quarters? *Ans.* 243 inches.

28. How many square inches in 10 square feet? *Ans.* 1440 square inches.

29. In 3 square miles, how many square rods? *Ans.* 307200*P.*

30. In 3 acres, 27 rods, how many square feet? *Ans.* $138030\frac{3}{4}$ square feet.

31. In 26025 square feet, how many square roods? *Ans.* 2*R.* 15*P.* $161\frac{1}{4}$ *sq. ft.*

32. In 70000 square links, how many square chains? *Ans.* 7 square chains.

33. How many square links in 5 acres?

Ans. 500000 square links.

34. In 17 cords of wood, how many cubic feet?

Ans. 2176.

35. In 17 tons of round timber, how many cubic inches?

Ans. 1175040.

36. Reduce 17900345 cubic inches to tons of hewn timber. *Ans*. 207 Tons, 8 cubic feet, 1721 cubic inches.

37. In 1000 cord feet of wood, how many cords?

Ans. 125.

38. In 19 cubic feet, how many cubic inches?

Ans. 32832.

39. In 16 hogsheads of wine, how many gills?

Ans. 32256.

40. In 10000 gills of wine, how many barrels?

Ans. 9 barrels 29 gallons.

41. Reduce 2 pipes, 7 barrels, 3 quarts of wine, to pints. *Ans*. 3786.

42. Reduce 31752 gills of wine to barrels.

Ans. 31 barrels 15 gallons 3 quarts.

43. Reduce 201600 gills to tuns of wine.

Ans. 25 tuns.

44. Reduce 11 hogsheads of beer to pints.

Ans. 4752.

45. In 100000 pints of beer, how many hogsheads?

Ans. 231 hogsheads 26 gallons.

46. In 10 hogsheads 1 quart 1 pint of beer, how many pints? *Ans*. 4323.

47. In 36 bushels, how many pints? *Ans*. 2304.

48. In 25 chaldrons 29 bushels, how many quarts?

Ans. 29728.

49. In 10000 pints, how many chaldrons?

Ans. 4*ch*. 12*bu*. 1*pk*.

50. In 1597 quarts, how many bushels?

Ans. 48*bu*. 3*pk*. 5*qt*.

In 30 days, how many seconds?

Ans. 2592000*sec*.

In 19 years of 365¼ days each, how many hours?

Ans. 166554

53. In 25 years 6 days, how many seconds?
Ans. 789458400*sec.*

54. How many days from the birth of Christ to Christmas, 1843, allowing the years to consist of 365 days 6 hours? *Ans.* 673155 days 18 hours.

55. A person was born May 3, 1795. How many days old was he May 3, 1821, paying particular attention to the order of leap year? *Ans.* 9496 days.

56. Suppose a person was born Feb. 29, 1796; how many birthdays will he have seen on Feb. 29, 1844, not counting the day on which he was born? *Ans.* 11.

57. In 3 signs 18 degrees, how many seconds?
Ans. 388800″.

58. In 6 signs 9 degrees, how many degrees?
Ans. 189°.

59. In 1000′ how many degrees? *Ans.* 16° 40′.

60. In 10000″ how many degrees? *Ans.* 2° 46′ 40″.

61. Reduce 45° 45′ 35″ to seconds. *Ans.* 164735″.

62. In 1000 things, how many dozen?
Ans. 83 dozen and 4 over.

63. How many buttons in 6⅓ dozen? *Ans.* 76.

64. In 80000 tacks, how many gross?
Ans. 555 gross 6 dozen and 8.

65. In three score and ten years, how many years?
Ans. 70.

66. In 15 quires of paper, how many sheets?
Ans. 360.

67. In a ream of paper, how many sheets? *Ans.* 480

ADDITION OF DENOMINATE NUMBERS.

64. If we wish to find the sum of £6 5*s.* 3*d.* 1 *far.*, £7 1*s.* 10*d.* 2 *far.*, £1 13*s.* 5*d.*, £4 18*s.* 0*d.* 2 *far.*, we proceed as follows:

OPERATION.

£	s.	d.	far.
6	5	3	1
7	1	10	2
1	13	5	0
4	18	0	2
£19	18s.	7d.	1 far.

Placing the quantities directly under each other, we add up the column of farthings, which we find to be 5. But we know that 5 farthings are equivalent to 1 penny and 1 farthing, we therefore write down the 1 farthing, and carry the penny into the next column, whose sum thus becomes 19 pence, which is the same as 1 shilling and 7 pence; we write down the 7 pence, and carry the shilling to the column of shillings, its sum then becomes 38 shillings, which is the same as 1 pound and 18 shillings; we write down the 18 shillings, and carry the pound into the next column of pounds, its sum then becomes 19 pounds; and since pounds is the highest denomination, we write down the whole.

From this example it will not be difficult to deduce this general

RULE.

I. Place the numbers so that those of the same denomination may stand directly under each other, and draw a line beneath them.

II. Add up the numbers in the lowest denomination, and find by reduction, how many units of the next higher denomination are contained in this sum; set down the remainder under its proper column, and carry those units to the next denomination, which add up as before.

III. Proceed thus through all the denominations to the highest, which last sum must be set down entire.

How do you place denominate numbers which are to be added? Which do you first add? Having added the column of lowest denominations, explain the subsequent work.

EXAMPLES.

£	s.	d.
7	13	3
3	5	10½
6	18	7
0	2	5¾
4	0	3
17	15	4½
39	15	9¾

£	s.	d.
11	0	5½
2	4	4
0	5	6¼
1	3	4
10	10	10
25	4	5¾

£	s.	d.
5	5	5
8	1	7¾
2	0	1¼
13	0	11¾
6	6	6
34	14	8

TROY WEIGHT.

lb.	oz.	pwt.	gr.
10	10	10	10
0	2	0	23
3	0	17	0
2	2	1	0
1	0	2	20
17	3	12	5

lb.	oz.	pwt.	gr.
6	5	4	1
1	11	19	13
0	3	0	4
8	9	1	2
4	4	18	0
21	10	2	20

lb.	oz.	pwt.	gr.
7	3	0	5
11	2	17	22
40	0	0	20
2	10	15	17
0	6	18	16
61	11	13	8

APOTHECARIES' WEIGHT.

℔	℥	ʒ	℈	gr.
8	10	7	2	19
10	0	6	0	10
0	1	2	1	15
5	1	2	1	15
8	0	5	1	13
32	3	0	2	12

℔	℥	ʒ	℈
2	11	6	0
10	8	3	1
14	10	2	2
0	6	5	0
7	5	4	1
36	6	5	1

ʒ	℈	gr.
1	0	18
2	1	15
3	2	13
4	0	0
6	1	7
18	0	13

AVOIRDUPOIS WEIGHT.

ton.	cwt.	qr.	lb.	oz.	dr.
10	18	2	25	15	1
1	15	0	0	14	15
12	0	1	3	0	10
0	13	0	27	1	11
2	2	2	0	7	8
27	9	3	1	7	13

cwt.	qr.	lb.	oz.
4	3	20	5
5	0	12	3
1	2	0	8
0	3	25	13
1	2	20	10
14	0	23	7

LONG MEASURE.

L.	*mi.*	*fur.*	*rd.*	*yd.*
1	2	6	37	4
6	0	0	30	5
0	1	4	0	3
2	0	1	1	0
3	2	0	25	1
14	0	5	15	2

rd.	*yd.*	*ft.*	*in.*
10	4	2	8
1	3	0	5
8	2	1	6
1	1	0	4
0	2	1	9
22	3	0	8

CLOTH MEASURE.

yd.	*qr.*	*na.*
15	1	2
13	0	3
20	2	2
0	3	0
8	1	1
58	1	0

E. Fl.	*qr.*	*na.*
3	2	3
15	1	2
9	2	0
8	0	1
10	0	0
47	0	2

E. E.	*qr.*	*na.*
4	2	2
10	1	1
9	2	0
13	0	2
15	1	1
52	2	2

SQUARE MEASURE.

Sq. yd.	*Sq. ft.*	*Sq. in.*
100	8	130
50	0	100
10	5	0
0	8	143
13	2	8
175	7	93

M.	*A.*	*R.*	*P.*
0	100	1	30
10	600	2	10
8	40	1	12
0	0	3	2
4	4	0	20
23	106	0	34

SOLID, OR CUBIC MEASURE.

S. yd.	*S. ft.*	*S. in.*
4	26	1000
1	10	1541
0	20	80
10	17	11
8	25	59
26	18	963

C.	*S. ft.*
10	120
8	100
2	80
0	119
12	6
35	41

C.	*Cord ft.*
3	7
10	4
12	1
8	6
15	3
50	5

WINE MEASURE.

hhd.	*gal.*	*qt.*	*pt.*
4	30	3	1
10	25	0	1
25	0	2	0
0	60	0	1
13	45	3	0
54	36	1	1

tun.	*pi.*	*hhd.*	*gal.*	*qt.*	*pt.*	*gi.*
1	1	1	37	3	1	3
10	0	0	50	0	1	2
11	0	1	13	1	0	1
4	1	0	25	2	0	0
8	0	1	18	0	1	3
36	0	1	19	0	1	1

ALE, OR BEER MEASURE.

hhd.	*gal.*	*qt.*	*pt.*
2	50	3	1
10	30	1	0
11	25	0	1
25	1	1	0
6	52	3	1
56	52	1	1

bar.	*gal.*	*qt.*
10	30	1
6	20	0
1	5	2
10	0	3
4	35	1
33	19	3

DRY MEASURE.

ch.	*bu.*	*pk.*	*qt.*	*pt.*
1	30	3	7	1
0	35	2	3	0
10	19	1	0	1
5	10	2	4	0
4	4	0	5	1
22	28	2	4	1

bu.	*pk.*	*qt.*	*pt.*
10	1	1	1
2	3	6	0
5	2	3	0
8	0	0	1
15	2	4	0
42	1	7	0

TIME.

da.	*hr.*	*m.*	*sec.*
15	18	50	49
1	13	59	59
4	23	0	2
10	11	1	4
0	2	10	15
32	21	2	9

wk.	*da.*	*hr.*	*m.*	*sec.*
1	2	13	40	30
2	6	10	8	3
0	5	22	55	45
2	3	4	1	15
1	2	4	5	0
8	6	6	50	33

CIRULAR MEASURE, OR MOTION.

c.	s.	°	′	″	s.	°	′	°	′	″
1	8	25	40	35	1	25	2	13	10	19
0	11	1	2	43	0	18	50	1	40	35
1	0	29	59	0	2	5	39	2	48	39
0	1	10	13	5	0	4	4	0	30	40
0	2	5	4	3	4	15	10	10	45	45
4	0	11	59	26	9	8	45	28	55	58

SUBTRACTION OF DENOMINATE NUMBERS.

65. If we wish to subtract £15 13s. 10d. from £20 5s. 8d., we proceed as follows:

OPERATION.

£	s.	d.
20	5	8
15	13	10
4	11	10

We place the terms of the subtrahend directly under the corresponding terms of the minuend, and draw a line underneath.

Commencing with the pence, we see that we can not subtract 10d. from 8d., we therefore increase 8d. by 12d., making 20d.; then subtracting 10d., we have remaining 10d. Carrying one to the 13s., it becomes 14s., this can not be subtracted from 5s., we therefore increase the 5s. by 20s., making it 25s. Now subtracting 14s., we have remaining 11s. Carrying one to £15 it becomes £16, which taken from £20 leaves £4.

Hence, we have this general

RULE.

I. Place the less number below the greater, so that those parts of the same denomination may stand directly under each other; draw a line below them.

II. Begin at the right, and subtract each number or part in the lower line from the one directly above it, and set the remainder below.

III. If any number in the lower line is greater than the one above it, add so many to the upper number as make one of the next higher denomination; then subtract the lower

number from the upper one thus increased, and set down the remainder. Carry the unit borrowed to the next number in the lower line; after which subtract this number from the one above it, as before; and thus proceed till the whole is finished.

PROOF

The proof is the same as in subtraction of simple numbers.

EXAMPLES.

	£	s.	d.
From	4	5	3½
Take	1	10	8
Remainder	2	14	7½
Proof	4	5	3½

yd.	qr.	na.
18	3	2
10	0	3
8	2	3
18	3	2

T.	cwt.	qr.	lb.	oz.	dr.
13	18	1	20	0	13
10	0	3	21	12	0
3	17	1	26	4	13

A.	R.	P.
69	3	25
10	0	38
59	2	27

℔	℥	ʒ	℈	gr.
24	7	2	1	16
16	10	3	2	17
7	8	6	1	19

L.	mi.	fur.	rd.
16	2	7	39
5	0	7	8
11	2	0	31

E. Fr.	qr.	na.
10	5	0
5	1	3
5	3	1

ch.	bu.	pk.	qt.	pt.
30	10	1	1	0
10	8	3	6	1
20	1	1	2	1

tun.	pi.	hhd.	gal.	qt.
10	1	1	50	1
1	0	0	60	3
9	1	0	52	2

da.	hr.	m.	sec.
100	10	0	1
60	0	40	45
40	9	19	16

yr.	mo.	wk.	da.
17	8	3	1
4	1	2	6
13	7	0	2

mi.	fur	rd.
60	0	0
40	7	39
19	0	1

C.	S. ft.
45	126
10	127
34	127

C.	Cord ft.
100	6
80	7
19	7

£	s.	d.
50	0	1
30	10	10
19	9	3

EXERCISES IN ADDITION AND SUBTRACTION.

1. Bought 20 yards of broadcloth for £18 5*s*. 3*d*., 30 pounds of feathers for £8 2*s*. 4*d*., 100 yards carpeting for £45 17*s*. 8*d*., 10 pieces of cotton cloth for £8 18*s*. 1*d*., 50 yards of calico for £2 0*s*. 10*d*. What was the cost of the whole? *Ans*. £83 4*s*. 2*d*.

2. Bought four hogsheads of sugar, weighing as follows: 1st weighed 8*cwt*. 1*qr*. 23*lb*. 10*oz*.; 2d weighed 9*cwt*. 2*qr*. 0*lb*. 3*oz*.; 3d weighed 10*cwt*. 0*qr*. 0*lb*. 8*oz*.; 4th weighed 8*cwt*. 3*qr*. 27*lb*. How much did the four weigh? *Ans*. 36*cwt*. 3*qr*. 23*lb*. 5*oz*.

3. A man owns three farms; the first contains 69 acres 3 roods 10 rods, the second contains 100 acres 5 rods, the third contains 150 acres 2 roods. How many acres are there in all? *Ans*. 320*A*. 1*R*. 15*P*.

4. Suppose a note given August 3d, 1838, to be paid November 10th, 1843. How long was the note on interest, if we count 30 days to the month? and how long if the time is accurately computed? 1st *Ans*. 5*yr*. 3*mo*. 7*da*.
2d *Ans*. 1925 days.

5. A person buys 15*cwt*. 3*qr*. 20*lb*. of sugar, and sells 10*cwt*. 0*qr*. 11*lb*. How much remains unsold?
Ans. 5*cwt*. 3*qr*. 9*lb*.

6. From a piece of cloth containing 37*yd*. 3*qr*. 2*na*., there has been taken at one time 6*yd*. 1*qr*., at another time 10*yd*. 3*qr*. 3*na*. How much then remains?
Ans 20*yd*. 2*qr*. 3*na*.

7. From a pile of wood containing 100 cords, I sold at one time 10*C.* 100*S.ft.*, at another time I sold 18*C.* 59*S.ft.* How many cords remain unsold?

Ans. 70*C.* 97*Sft.*

8. A farmer raises 100*bu.* 3*pk.* 2*qt.* of wheat from one field, 87*bu.* 1*pk.* 1*qt.* 1*pt.* from another field; he sells 53*bu.* to one person, and 37*bu.* 2*pk.* 1*qt.* to another person. How many bushels has he remaining?

Ans. 97*bu.* 2*pk.* 2*qt.* 1*pt.*

MULTIPLICATION OF DENOMINATE NUMBERS.

66. If we wish to multiply £13 5*s.* 10*d.* by 5, we proceed as follows:

OPERATION.

£	*s.*	*d.*
13	5	10
		5
66	9	2

First, we say 5 times 10*d.* is 50*d.*, which equals 4*s.* and 2*d.*; we set down the 2*d.* and reserve the 4*s.* to carry into the next column. We then say 5 times 5*s.* equals 25*s.*, to which adding the 4*s.* we have 29*s.*, which equals £1 9*s.*; we set down the 9*s.* and reserve the £1 to carry to the next denomination. Finally, we say 5 times £13 is £65, to which adding the £1, we have £66; this being the highest denomination, we set it down entire.

From this we conclude this general

RULE.

I. Set the multiplier under the lowest denomination of the multiplicand, and draw a line below it.

II. Multiply the number in the lowest denomination by the multiplier, and find how many units of the next higher denomination are contained in the product, setting down what remains. In like manner, multiply the number in the next denomination, and to the product carry or add the units before found, and find how many units of the next higher denomination are contained in this amount, which carry in like manner to the next product, setting down the overplus.

III. Proceed thus to the highest denomination proposed.

In multiplication of denominate numbers, where do you set the multiplier? Which denominate value do you first multiply? After finding in the product the number of units of next higher order and also what remains, where do you place the remainder; and what do you do with the units of next superior order? Repeat the rest of the Rule.

EXAMPLES.

(1.)

£	s.	d.
10	10	10
		3
31	12	6

(2.)

T.	cwt.	qr.	lb.	oz.	dr.
	8	0	2	4	5
					6
2	8	0	13	9	14

3. In 3 hogsheads of sugar, each containing 10*cwt.* 3*qr.* 5*lb.*, how many hundred weight? *Ans.* 32*cwt.* 1*qr.* 15*lb.*

4. How much cloth will it take for 7 suits of clothes, if each suit require 7*yd.* 3*qr.* 1*na.*?

Ans. 54*yd.* 2*qr.* 3*na.*

5. How much wood can a man draw in 13 loads, if he draw 1*C.* 19*S.ft.* at each load? *Ans.* 14*C.* 119*S.ft.*

6. How long will it take a man to saw 6 cords of wood, if he employ 7*hr.* 30*m.* 45*sec.* to saw one cord, allowing 10 working hours for each day?

Ans. 4*da.* 5*hr.* 4*m.* 30*sec.*

7. The circumference of a wheel is 15 feet 2 inches. What distance will this wheel measure on the ground, if it is rolled over 365 times? *Ans.* 1*mi.* 255*ft.* 10*in.*

8. Allowing the year to consist accurately of 365 days 5 hours 48 minutes 48 seconds, what will be the true length of 1843 years? *Ans.* 673141*da.* 9*hr.* 58*m.* 24*sec.*

When the multiplier is a composite number, we may, as in simple numbers, multiply successively by the component parts.

9. What will 35*cwt.* of cheese cost, at 15*s.* 6*d.* per hundred weight?

OPERATION.

£	s.	d.	
	15	6	cost of 1*cwt.*
		5	
3	17	6	cost of 5*cwt.*
		7	×7
27	2	6	cost of 35*cwt.*

10. How much brandy in 84*pi.*, each containing 128*gal.* 2*qt.* 1*pt.* 3*gi.*? *Ans.* 10812*gal.* 1*qt.* 1*pt.*

11. In 21 loads of wood, each 1*C.* 1*C.ft.*, how many cords? *Ans.* 23*C.* 5*C.ft.*

12. Suppose the piston rod of a steam engine to move 3*ft.* 4*in.* 1*b.c.* at each stroke. Through what distance will it move in making 1000 strokes?

Ans. 3361*ft.* 1*in* 1*b c.*

13. Bought as follows:

lb.			*s.*	*d.*	
18	of green tea	at	12	3	per pound.
12	of raisins	at	1	2	"
27	of loaf sugar	at	1	4	"
15	of English currants	at	2	3	"
14	of citron	at	3	6	"

What is the amount of the whole purchase?

Ans. £17 13*s.* 3*d.*

DIVISION OF DENOMINATE NUMBERS.

67. Let it be required to divide £100 10*s.* 3*d.* equally among 17 men.

OPERATION.

17)£100 10*s*. 3*d*.(£5
85
15
20
17)310(18*s*.
17
140
136
4
12
17)51(3*d*.
51

Collecting, we have £5 18*s*. 3*d*.

EXPLANATION.

First, we say 17 in £100 is contained 5 times and £15 remaining; and since this £15 is yet to be divided among the 17 men, as well as the 10*s*., we reduce it to shillings, and add the 10*s*. to it; we thus find 310*s*., which we proceed to divide among the 17 men; we find 17 to be contained 18 times in 310*s*. with 4*s*. remainder. We reduce this 4*s*. to pence, and add the 3*d*., and thus obtain 51*d*., which divided among the 17 men, gives 3*d*. each.

Had the divisor been one of the nine digits, the work might have been performed by short division.

We therefore have this general

RULE.

I. Place the divisor on the left of the dividend, as in simple division. Begin at the left-hand, and divide the number of the highest denomination by the divisor. Reduce the remainder, if any, to the next lower denomination, to which add the number of the dividend expressing that denomination, and then divide the sum by the divisor.

II. Proceed in the same way for all the denominations. If there is a remainder in the last division, place it over the divisor, as in simple numbers. Each quotient will be of the same denomination as its dividend.

Having placed the divisor as in Simple Division, how do you proceed? When, in dividing any particular denomination, there is a remainder, how do you dispose of it? Of what denomination will the respective quotients be?

EXAMPLES.

(1.)

	yd.	*qr.*	*na.*
7)	25	3	1
	3	2	3

(2.)

	cwt.	*qr.*	*lb.*	*oz.*	*dr.*
9)	27	3	26	13	9
	3	0	12	5	1

(3.)

lb. *oz.* *pwt.* *gr.*
13)10 8 16 3(0*lb.*
12
13)128(9*oz.*
117
11
20
13)236(18*pwt.*
13
106
104
2
24
13)51(3$\frac{12}{13}$*gr.*
39
12 remainder.

(4.)

```
      mi. fur. rd.  yd.  ft.
23)100   4   30   1½   2(4mi.
    92
     8
     8
23)68(2fur.
   46
   22
   40
23)910(39rd.
   69
   220
   207
    13
    5½
23)73 (3yd.
   69
    4
    3
23)14(0ft.
   12
23)168( 7 7/23in.
   161
     7 remainder.
```

5. Divide 10*tuns* 2*hhd.* 17*gal.* 2*pt.* by 67

Ans. 39*gal.* 6*pt.*

6. Divide 51*A.* 1*R.* 11*P.* by 51. *Ans.* 1*A.* 0*R.* 1*P.*

7. Divide 4*gal.* 2*qt.* by 144. *Ans.* 1*gi.*

8. Divide £113 13*s.* 4*d.* by 31. *Ans.* £3 13*s.* 4*d.*

9. Divide 673141*da.* 9*hr.* 58*m.* 24*sec.* by 1843.

Ans. 365*da.* 5*h.* 48*m.* 48*sec.*

10. Divide 1*mi.* 255*ft.* 10*in.* by 365. *Ans.* 15*ft.* 2*in.*

When the divisor is a composite number, we may divide in succession by each of its component parts.

11. Bought 15 sheep for £5 12s. 6d. How much did one sheep cost?

FIRST OPERATION.		SECOND OPERATION.	
£ s. d.		£ s. d.	
3)5 12 6	cost of 15 sheep.	5)5 12 6	cost of 15 sheep.
5)1 17 6	cost of 5 sheep.	3)1 2 6	cost of 3 sheep.
0 7 6	cost of 1 sheep.	0 7 6	cost of 1 sheep.

From this example we see that it makes no difference which component factor is first used.

12. If 24*yd.* of cloth cost £18 6*s.*, how much is that per yard? *Ans.* 15*s.* 3*d.*

13. From a piece of cloth containing 128*yd.* 1*qr.*, a tailor made 18 coats, which took one third of the whole piece. How many yards did each coat contain?

Ans. 2*yd.* 1*qr.* 2*na.*

QUESTIONS EXERCISING THE FOUR PRECEDING RULES.

1. Twenty-four men agree to construct 7*mi.* 1*fur.* 24*rd.* of road; after completing $\frac{1}{3}$ of it, they employ 8 more men. What distance does each man construct before and after the 8 men were employed?

Ans. { 16*rd.* before.
1*fur.* 20*rd.* after.

2. A silversmith has seven tea-pots, each weighing 1*lb.* 3*oz.* 13*pwt.* 11*gr.* What is the whole weight?

Ans. 9*lb.* 1*oz.* 14*pwt.* 5*gr.*

3. A farmer has 1000 bushels of apples, which he puts into 350 barrels. How many does each barrel hold?

Ans. 2*bu.* 3*pk.* $3\frac{3}{7}$ *qt.*

4. If it require 1 sheet of paper to print 24 pages of a book, how many reams, allowing 18 quires to the ream, will it take to print 3000 copies, of 250 pages each?

Ans. 72 reams 6 quires 2 sheets.

5. An estate worth £2570 is to be divided as follows: the widow has one third of the whole, the remainder is to be divided equally between seven children. How much

does the widow receive, and how much does each child have?

Ans. { The widow has £856 13*s.* 4*d.*
Each child has £244 15*s.* 2*d.* $3\frac{3}{7}$*far.*

6. Divide 100 acres 3 roods 8 rods of land between four persons, A, B, C, and D, so that A shall have one sixth of the whole, B one fourth of the remainder, C one third of what then remains, and D the rest. How much does each one have?

Ans.				
	A had	16*A.*	3*R.*	8*P.*
	B had	21	0	0
	C had	21	0	0
	D had	42	0	0

7. A, B, C, and D, having 13*cwt.* 1*qr.* 4*lb.* of sugar, they agree to divide it as follows: A is to have one half of the whole, B is to have one third of the remainder, C is to have one fourth of what then remains, and D is to take what is left. What were their respective portions?

Ans.				
	A had	6*cwt.*	2*qr.*	16*lb.*
	B had	2	0	24
	C had	1	0	12
	D had	3	1	8

---

68. On page 49 we gave definitions of the different kinds of vulgar fractions. We are now prepared to explain fully the various operations which may be performed by the aid of fractions; and shall commence with the

REDUCTION OF FRACTIONS.

Since the value of a fraction is the quotient arising from dividing the numerator by the denominator, we may infer the following

PROPOSITIONS.

I. That, multiplying the numerator by any number, is the same as multiplying the value of the fraction by the same number.

II. That, multiplying the denominator by any number, is the same as dividing the value of the fraction by the same number.

III. That, multiplying both numerator and denominator by any number, does not alter the value of the fraction.

IV. That, dividing the numerator by any number, is the same as dividing the value of the fraction by the same number.

V. That, dividing the denominator by any number, is the same as multiplying the value of the fraction by the same number.

VI. That, dividing both numerator and denominator by the same number, does not alter its value.

GREATEST COMMON DIVISOR.

69. The greatest common divisor of two or more numbers, is the greatest number which will divide them without any remainder.

Let us endeavor to find the greatest common divisor of 360 and 276.

It is evident that the greatest divisor can not exceed the less number, 276. Now, since 276 is divisible by itself, it will be the greatest common divisor, provided it will divide the other number, 360. If we attempt to divide 360 by 276, we shall find the quotient 1, and the remainder 84. It is likewise evident, that the greatest number which will divide two numbers, will also divide their difference. We might also infer that any number of times the less number being subtracted from the greater, the remainder would be divisible by their greatest common divisor. Hence, the greatest divisor of 276 and 84, is also the greatest common divisor of 360 and 276. Again, dividing 276 by 84, we find 3 for a quotient and 24 for the remainder. By the same reasoning as just employed, we discover that the greatest common measure of 84 and 24, is also the greatest common measure of 276 and 84,

and consequently of 360 and 276. Now, dividing 84 by 24, we find the quotient 3, and remainder 12. Finally, dividing 24 by 12, and we find it is contained exactly twice; so that the greatest common divisor of 24 and 12 is 12: consequently, 12 is the greatest common divisor of 360 and 276. We will exhibit in one point of view the above.

OPERATION.

```
276)360(1
    276
    ---
     84)276(3
        252
        ---
         24)84(3
            72
            --
            12)24(2
               24
               --
                0
```

Hence, to find the greatest common divisor of two numbers, we deduce this

RULE.

Divide the greater number by the less, then the less number by the remainder; thus continue to divide the last divisor by the last remainder, until there is no remainder. The last divisor will be the greatest common divisor.

NOTE.—When there are more than two numbers whose greatest common divisor is required, we must first find the greatest common divisor of any two, and then find the greatest common divisor of this divisor thus found, and one of the remaining numbers; and thus continue until all the different numbers have been used.

What is the greatest common divisor of two or more numbers? Repeat the rule for finding the greatest common divisor of two numbers. How do you proceed when there are more than two numbers?

EXAMPLES.

Find the greatest common divisor of 592 and 999.

OPERATION.

```
592)999(1
    592
    407)592(1
        407
        185)407(2
            370
             37)185(5
                185
                  0
```

›m which we obtain 37 for the greatest common ›r of 592 and 999.

What is the greatest common divisor of 492, 744, 06?

› first find the greatest common divisor of 492 and ›y the following

OPERATION.

```
492)744(1
    492
    252)492(1
        252
        240)252(1
            240
             12)240(20
                240
                  0
```

›fore, the greatest common divisor of 492 and 744

ain, proceeding with 12 and 906, we have the fol- g

OPERATION.

```
12)906(75
   900
   ---
     6)12(2
       12
       --
        0
```

We thus find 6 to be the greatest common divisor of 12 and 906, and consequently of the three numbers 492, 744, and 906.

3. What is the greatest common divisor of 315 and 405? *Ans.* 45.

4. What is the greatest common divisor of 1825 and 2655? *Ans.* 5.

5. What is the greatest common divisor of 506 and 308? *Ans.* 22.

6. What is the greatest common divisor of 404 and 364? *Ans.* 4.

7. What is the greatest common divisor of 246, 372, and 522? *Ans.* 6.

70. We are now prepared to proceed to the reduction of fractions.

We know from PROP. VI., ART. **68**, that we can divide both numerator and denominator of a fraction by any number without altering its value. If we divide by the greatest common divisor, the resulting fraction will be in its lowest terms.

Therefore, to reduce a fraction to its lowest terms, we have this

RULE.

Divide both numerator and denominator by their greatest common divisor.

How do you reduce a fraction to its lowest terms?

EXAMPLES.

1. Reduce $\frac{592}{999}$ to its lowest terms.

We have already found (Ex. 1, ART. **69**) the greatest common divisor of 592 and 999 to be 37. Dividing both

these terms by 37, we find 16 and 27 for quotients; hence, $\frac{592}{999}$, when reduced to its lowest terms, becomes $\frac{16}{27}$.

2. Reduce $\frac{1049}{8392}$ to its lowest terms. *Ans.* $\frac{1}{8}$
3. Reduce $\frac{30}{60}$, $\frac{60}{180}$, $\frac{45}{180}$, to their lowest terms. *Ans.* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
4. Reduce $\frac{315}{405}$ to its lowest terms. *Ans.* $\frac{7}{9}$.
5. Reduce $\frac{172}{1118}$ to its lowest terms. *Ans.* $\frac{2}{13}$.
6. Reduce $\frac{275}{440}$ to its lowest terms. *Ans.* $\frac{5}{8}$.
7. Reduce $\frac{99}{9999}$ to its lowest terms. *Ans.* $\frac{1}{101}$.
8. Reduce $\frac{4104}{4560}$ to its lowest terms. *Ans.* $\frac{9}{10}$.
9. Reduce $\frac{33522}{41223}$ to its lowest terms. *Ans.* $\frac{74}{91}$.
10. Reduce $\frac{68760}{68103}$ to its lowest terms. *Ans.* $\frac{8}{9}$.

We may frequently discover numbers, by inspection, which will divide both numerator and denominator without a remainder. When this is the case, we need not resort to the rule for obtaining the greatest common measure, until we have divided by such factors.

11. Reduce $\frac{5184}{6912}$ to its lowest terms.

$$\frac{5184}{6912} \overset{\div 4}{=} \frac{1296}{1728} \overset{\div 4}{=} \frac{324}{432} \overset{\div 4}{=} \frac{81}{108} \overset{\div 3}{=} \frac{27}{36} \overset{\div 3}{=} \frac{9}{12} \overset{\div 3}{=} \frac{3}{4}.$$

12. Reduce $\frac{162}{324}$ to its lowest terms.

$$\frac{162}{324} \overset{\div 2}{=} \frac{81}{162} \overset{\div 3}{=} \frac{27}{54} \overset{\div 3}{=} \frac{9}{18} \overset{\div 3}{=} \frac{3}{6} \overset{\div 3}{=} \frac{1}{2}.$$

13. Reduce $\frac{360}{420}$ to its lowest terms. *Ans.* $\frac{6}{7}$.
14. Reduce $\frac{1386}{1638}$ to its lowest terms. *Ans.* $\frac{11}{13}$.
15. Reduce $\frac{14175}{22050}$ to its lowest terms. *Ans.* $\frac{9}{14}$.
16. Reduce $\frac{472500}{787500}$ to its lowest terms. *Ans.* $\frac{3}{5}$.

71. To reduce an improper fraction to a mixed number.

RULE.

Divide the numerator by the denominator, the quotient will be the integral part of the mixed number. The remainder being placed over the denominator of the improper fraction, will form the fractional part.

Repeat the Rule for reducing an improper fraction to a mixed number.

This rule is obviously correct, since the value of a fraction is the numerator divided by the denominator.

EXAMPLES.

1. Reduce $\frac{60}{25}$ to a mixed number.

Dividing 60 by 25, we find 2 for the quotient, and 10 for the remainder; therefore, $\frac{60}{25}$ is equal to the mixed number $2\frac{10}{25}=2\frac{2}{5}$.

2. Reduce $\frac{45}{2}$, $\frac{75}{4}$, $\frac{80}{7}$, $\frac{90}{12}$, to mixed numbers. *Ans.* $22\frac{1}{2}$, $18\frac{3}{4}$, $11\frac{3}{7}$, $7\frac{1}{2}$.

3. Reduce $\frac{50}{6}$, $\frac{80}{9}$, $\frac{37}{10}$, to mixed numbers. *Ans.* $8\frac{1}{3}$, $8\frac{8}{9}$, $3\frac{7}{10}$.

4. Reduce $\frac{131}{7}$ to a mixed number. *Ans.* $18\frac{5}{7}$.

5 Reduce $\frac{29817}{365}$ to a mixed number. *Ans.* $81\frac{252}{365}$.

72. To reduce a mixed number to an equivalent improper fraction, we have this

RULE.

Multiply the integral part of the mixed number by the denominator of the fractional part, to the product add the numerator of the fractional part, the sum will be the numerator of the improper fraction; under which place the denominator of the fractional part.

Repeat the Rule for reducing a mixed number to an improper fraction.

This rule is obviously correct, since it is the reverse of the rule under ART. **71**, where a reverse operation was required to be performed.

EXAMPLES.

1. Reduce $4\frac{1}{2}$ to an improper fraction. *Ans.* $\frac{9}{2}$.

2. Reduce $3\frac{1}{3}$, $7\frac{2}{5}$, $8\frac{7}{9}$, to improper fractions. *Ans.* $\frac{10}{3}$, $\frac{37}{5}$, $\frac{79}{9}$.

3. Reduce $8\frac{1}{3}$, $18\frac{3}{4}$, $11\frac{3}{7}$, $7\frac{1}{2}$, to improper fractions. *Ans.* $\frac{25}{3}$, $\frac{75}{4}$, $\frac{80}{7}$, $\frac{15}{2}$.

4. Reduce $81\frac{252}{365}$ to an improper fraction. *Ans.* $\frac{29817}{365}$.

73. Let us endeavor to reduce the compound fraction $\frac{3}{4}$ of $\frac{7}{11}$ to an equivalent simple fraction.

$\frac{1}{4}$ of $\frac{7}{11}$ can be obtained by dividing the value of the fraction $\frac{7}{11}$ by 4, which (by PROP. II. ART. **68**), can be effected by multiplying the denominator by 4; therefore, $\frac{1}{4}$ of $\frac{7}{11}$ equals $\frac{7}{4\times 11}$.

Again, $\frac{3}{4}$ of $\frac{7}{11}$ is obviously three times as great as $\frac{1}{4}$ of $\frac{7}{11}$; therefore, to obtain $\frac{3}{4}$ of $\frac{7}{11}$, we must multiply $\frac{7}{4\times 11}$ by 3, which (by PROP. I. ART. **68**), can be done by multiplying the numerator by 3; hence, we have $\frac{3}{4}$ of $\frac{7}{11}=\frac{3\times 7}{4\times 11}=\frac{21}{44}$.

Therefore, to reduce compound fractions to their equivalent simple ones, we have this

RULE.

Consider the word OF, *which connects the fractional parts, as equivalent to the sign of multiplication. Then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; always observing to reject or cancel such factors as are common to the numerators and denominators, which is the same as dividing both numerator and denominator by the same quantity, and (by* PROP. VI. ART. **68**), *does not change the value of the resulting fraction.*

Repeat the Rule for reducing a compound fraction to a simple one.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{15}$ of $\frac{5}{12}$ to its equivalent simple fraction.

Substituting the sign of multiplication for the word *of*, we get $\frac{1}{2}\times\frac{3}{4}\times\frac{8}{15}\times\frac{5}{12}$. First, cancelling the 8 of the numerator against the 2 and 4 of the denominator, by drawing a line across them, we get

$$\frac{1}{\cancel{2}}\times\frac{3}{\cancel{4}}\times\frac{\cancel{8}}{15}\times\frac{5}{12}.$$

Again, cancelling the 3 and 5 of the numerator against the 15 of the denominator, we finally obtain

$$\frac{1}{\cancel{2}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{8}}{\cancel{15}}\times\frac{\cancel{5}}{12}=\frac{1}{12}.$$

2. Reduce $\frac{3}{7}$ of $\frac{14}{35}$ of $\frac{7}{8}$ of $\frac{4}{9}$ of $\frac{5}{11}$ to its simplest form.

First, cancelling the 7 and 5 of the numerator against the 35 of the denominator, we get

$$\frac{3}{7}\times\frac{14}{\cancel{35}}\times\frac{\cancel{7}}{8}\times\frac{4}{9}\times\frac{\cancel{5}}{11}.$$

Again, cancelling the 7 of the denominator against a part of the 14 of the numerator, and the 3 of the numerator against a part of the 9 of the denominator, we obtain

$$\frac{\cancel{3}}{\cancel{7}}\times\frac{\overset{2}{\cancel{14}}}{\cancel{35}}\times\frac{\cancel{7}}{8}\times\frac{4}{\underset{3}{\cancel{9}}}\times\frac{\cancel{5}}{11}.$$

Finally, cancelling the 2 and 4 of the numerator against the 8 of the denominator, we get

$$\frac{\cancel{3}}{\cancel{7}}\times\frac{\overset{\cancel{2}}{\cancel{14}}}{\cancel{35}}\times\frac{\cancel{7}}{\cancel{8}}\times\frac{\cancel{4}}{\underset{3}{\cancel{9}}}\times\frac{\cancel{5}}{11}=\frac{1}{33}.$$

NOTE.—We have written our fractions several times, in order the more clearly to exhibit the process of cancelling. But in practice, it will not be necessary to write the fractions more than once. It will make no difference which of the factors is first cancelled. When all the common factors have in this way been stricken out, the fraction will then appear in its lowest terms.

3. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{7}{9}$ to its simplest form.
Ans. $\frac{1}{3}$.

4. Reduce $\frac{3}{8}$ of $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{1}{7}$ to its simplest form.
Ans. $\frac{1}{28}$.

5. Reduce $\frac{3}{7}$ of $\frac{14}{15}$ of $\frac{10}{12}$ of $\frac{3}{7}$ of $\frac{14}{17}$ to its simplest form.
Ans. $\frac{2}{17}$.

6. Reduce $\frac{1}{2}$ of $2\frac{1}{2}$ of $3\frac{1}{3}$ of 6 to its simplest form.
Ans. 25.

74. To reduce fractions to a common denominator, we have this

RULE.

Reduce mixed numbers to improper fractions, and compound fractions to their simplest form. Then multiply each numerator by all the denominators except its own, for a new numerator, and all the denominators together for a common denominator.

Repeat this Rule.

It is obvious that this process will give the same denominator to each fraction, viz.: the product of all the denominators.

It is also obvious, that the values of the fractions will not be changed, since both numerator and denominator are multiplied by the same quantity, viz.: the product of all the denominators except its own.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{5}{3}$ of $\frac{2}{5}$, $\frac{3}{11}$, and $\frac{7}{9}$ of $\frac{2}{7}$, to equivalent fractions having a common denominator.

These fractions, when reduced to their simplest form, become

$$\frac{1}{2},\ \frac{2}{3},\ \frac{3}{11},\ \text{and}\ \frac{2}{9}.$$

The new numerator of first fraction is $1\times3\times11\times9=297$.

The new numerator of second fraction is $2\times2\times11\times9=396$.

The new numerator of third fraction is $3\times2\times3\times9=162$.

The new numerator of fourth fraction is $2\times2\times3\times11=132$.

The common denominator is $2\times3\times11\times9=594$.

Therefore, the fractions when reduced to a common denominator, are

$$\frac{297}{594},\ \frac{396}{594},\ \frac{162}{594},\ \text{and}\ \frac{132}{594}.$$

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, to equivalent fractions having a common denominator. *Ans.* $\frac{12}{24}$, $\frac{8}{24}$, $\frac{6}{24}$.

3. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, to equivalent fractions having a common denominator. *Ans.* $\frac{240}{360}$, $\frac{270}{360}$, $\frac{288}{360}$, $\frac{300}{360}$.

4. Reduce $\frac{1}{2}$ of $\frac{2}{3}$, $4\frac{1}{2}$, $5\frac{1}{3}$, to equivalent fractions having a common denominator. *Ans* $\frac{6}{18}$, $\frac{81}{18}$, $\frac{96}{18}$.

5 Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of 5, $7\frac{1}{3}$, $5\frac{1}{5}$, to equivalent fractions having a common denominator. *Ans.* $\frac{75}{30}$, $\frac{220}{30}$, $\frac{156}{30}$.

75. In most cases fractions may be reduced to equivalent ones having a smaller common denominator than is given by the above rule. Before showing how to find the *least common denominator* of fractions, it becomes necessary to show how to find

THE LEAST COMMON MULTIPLE.

A *multiple* of several numbers is such a number as can be divided by each of them without a remainder. Thus, 12, 24, 36, 48, &c., are multiples of 2, 3, 4, and 6, since each of them is divisible by 2, 3, 4, and 6. Any set of numbers may have an infinite number of multiples. In practice it is the *least* common multiple which is usually sought. In the above example, 12 is the least common multiple of 2, 3, 4, and 6.

The least common multiple of any set of numbers may be found by the following

RULE.

Write the numbers in a horizontal line, divide them by the least number which will divide two or more of them without a remainder; place the quotients with the undivided terms, if any, for a second horizontal line; proceed with this second line as with the first; and so continue until there are no two terms which can be divided. The continued product of the divisors and numbers in the last horizontal line will give the least multiple.

NOTE.—When there is no number which will divide two of the given numbers, their continued product must be taken for the least common multiple.

What is a multiple of several numbers? Mention some of the multiples of 2, 3, 4, and 6. Are the number of multiples of any set of numbers limited? Repeat the Rule for finding the least common multiple of any set of numbers. When there is no number which will divide two of the given numbers, how is the least multiple found?

EXAMPLES.

1. What is the least common multiple of 12, 16, and 24?

OPERATION.

2	12,	16,	24.
2	6,	8,	12.
2	3,	4,	6.
3	3,	2,	3.
	1,	2,	1.

Hence, $2 \times 2 \times 2 \times 3 \times 2 = 48$ is the least common multiple.

2. What is the least common multiple of 12, 15, 24?

OPERATION.

2	12,	15,	24.
2	6,	15,	12.
3	3,	15,	6.
	1,	5,	2.

Therefore, $2 \times 2 \times 3 \times 5 \times 2 = 120$ is the multiple sought.

3. What is the least common multiple of 11, 77, 88? *Ans.* 616.

4. What is the least common multiple of 37, 41? *Ans.* 1517.

5. What is the least common multiple of 24, 60, 45, 180? *Ans.* 360.

6. What is the least common multiple of 2, 4, 6, 8? *Ans.* 24.

7. What is the least common multiple of 3, 5, 7, 9? *Ans.* 315

8. What is the least common multiple of 2, 3, 4, 5, 6, 7, 8, 9? *Ans.* 2520.

76. We are now prepared to reduce fractions to their least common denominator by the following

RULE.

Reduce the fractions to their simplest form; then find the least common multiple of their denominators, (by Rule under Art. **75**,) *which will be their least common denominator. Divide this denominator by the respective denominators of the given fractions, and multiply the quotients by their respective numerators, and the respective products will be the new numerators.*

Repeat the Rule for reducing fractions to their least common denominator.

EXAMPLES.

1. Reduce $\frac{5}{12}$, $\frac{7}{16}$, $\frac{11}{24}$, to equivalent fractions having the least common denominator.

The least common multiple of the denominators 12, 16, 24, is 48=common denominator.

New numerator of first fraction $\frac{48}{12}\times 5=20$.
New numerator of second fraction $\frac{48}{16}\times 7=21$.
New numerator of third fraction $\frac{48}{24}\times 11=22$.

Hence, the fractions, when reduced to their least common denominator, become

$$\frac{20}{48}, \frac{21}{48}, \frac{22}{48}.$$

2. Reduce $\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{7}{12}$, $\frac{3}{20}$, $\frac{7}{15}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{15}{120}$, $\frac{18}{120}$, $\frac{56}{120}$.

3. Reduce $3\frac{1}{2}$, $4\frac{1}{3}$, $\frac{6}{5}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{105}{30}$, $\frac{130}{30}$, $\frac{36}{30}$.

4. Reduce $\frac{8}{9}$, $\frac{7}{15}$, $\frac{13}{20}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{160}{180}$, $\frac{84}{180}$, $\frac{117}{180}$.

5. Reduce $\frac{4}{15}$, $\frac{5}{11}$, $6\frac{3}{22}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{88}{330}$, $\frac{150}{330}$, $\frac{2025}{330}$.

6. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $3\frac{1}{4}$, and $\frac{1}{5}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{195}{60}$, $\frac{12}{60}$.

7. Reduce $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{7}$, $\frac{4}{21}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{21}{210}$, $\frac{70}{210}$, $\frac{30}{210}$, $\frac{40}{210}$.

8. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{20}$, to equivalent fractions having the least common denominator. *Ans.* $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$, $\frac{27}{60}$.

ADDITION OF FRACTIONS.

77. Suppose we wish to add $\frac{3}{7}$ and $\frac{4}{5}$. We know that so long as these fractions have different denominators, they can not be added; we will therefore reduce them to a common denominator. We thus obtain

$$\frac{3}{7}=\frac{15}{35}$$
$$\frac{4}{5}=\frac{28}{35}$$

Now, taking their sum we obtain

$$\frac{3}{7}+\frac{4}{5}=\frac{15}{35}+\frac{28}{35}=\frac{15+28}{35}=\frac{43}{35}=1\frac{8}{35}.$$

Hence, to add fractions, we have this

RULE.

Reduce the fractions to a common denominator, and take the sum of their numerators, under which place the common denominator, and it will give the sum required.

NOTE.—The labor will be the least when we reduce the fractions to their *least common denominator.*

EXAMPLES.

1. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$?

These fractions, when reduced to their least common denominator, are $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, and $\frac{2}{12}$, the sum of whose numerators is $6+4+3+2=15$. Hence we have

$$\frac{1}{2}+\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{15}{12}=\frac{5}{4}=1\frac{1}{4}.$$

2. What is the sum of $\frac{1}{6}$ and $\frac{1}{8}$? *Ans.* $\frac{7}{24}$.
3. What is the sum of $\frac{1}{5}$, $\frac{1}{10}$, $\frac{3}{20}$? *Ans.* $\frac{9}{20}$.
4. What is the sum of $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{12}$? *Ans.* $\frac{29}{24}=1\frac{5}{24}$.
5. What is the sum of $\frac{4}{15}$, $\frac{5}{12}$, $\frac{3}{10}$? *Ans.* $\frac{59}{60}$.
6. What is the sum of $\frac{3}{5}$, $\frac{4}{15}$, $\frac{7}{10}$, $\frac{9}{30}$? *Ans.* $\frac{56}{30}=1\frac{13}{15}$.

NOTE.—If any of the fractions are compound, they must first be reduced to simple fractions (by Rule under ART. **73**).

7. What is the sum of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{9}$, $\frac{1}{3}$ of $\frac{1}{4}$, and $\frac{3}{8}$?

These fractions, when reduced to their simplest forms, are $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{3}{8}$, which, when reduced to their least common denominator, become

$$\frac{4}{24},\ \frac{2}{24},\ \frac{9}{24}.$$

Hence, their sum is

$$\frac{4+2+9}{24}=\frac{15}{24}=\frac{5}{8}.$$

8. What is the sum of $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{10}{12}$, $\frac{2}{3}$ of $\frac{3}{4}$ of 6, and $\frac{1}{2}$ of $\frac{6}{7}$ of 3? *Ans.* $4\frac{13}{21}$.

9 What is the sum of $\frac{3}{5}$ of $\frac{5}{4}$ of 8, $\frac{1}{8}$ of $\frac{4}{5}$ of $\frac{10}{3}$, and $\frac{1}{8}$ of 16? *Ans.* $8\frac{1}{3}$

10. What is the sum of $\frac{1}{4}$ of $\frac{4}{3}$ of $\frac{3}{5}$, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{2}{5}$ of $\frac{5}{8}$? *Ans.* $\frac{7}{10}$.

11. What is the sum of $\frac{3}{8}$, $\frac{4}{7}$, $\frac{5}{9}$, and $\frac{3}{4}$? *Ans.* $\frac{1135}{504}=2\frac{127}{504}$.

12. What is the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{11}{12}$? *Ans.* $\frac{44}{12}=3\frac{2}{3}$.

13. What is the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$, and $\frac{19}{20}$? *Ans.* $\frac{78}{20}=3\frac{9}{10}$.

14. What is the sum of $3\frac{1}{2}$, $\frac{1}{4}$ of $\frac{1}{5}$, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$, and $\frac{1}{15}$? *Ans.* $\frac{241}{60}=4\frac{1}{60}$.

15. What is the sum of $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{6}{7}$, and $\frac{8}{9}$ of $\frac{9}{10}$? *Ans.* $\frac{257}{105}=2\frac{47}{105}$.

16. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$? *Ans.* $\frac{4609}{2520}=1\frac{2089}{2520}$.

SUBTRACTION OF FRACTIONS.

78. To subtract one fraction from another we have this

RULE.

Reduce the fractions to a common denominator, and subtract the numerator of the subtrahend from that of the minuend; place the common denominator under the difference.

Repeat this Rule.

EXAMPLES.

1. From $\frac{1}{2}$ subtract $\frac{1}{5}$.

Reducing these fractions to a common denominator, they become $\frac{5}{10}$ and $\frac{2}{10}$; subtracting the numerators we find $5-2=3$. Therefore we have

$$\frac{1}{2}-\frac{1}{5}=\frac{3}{10}.$$

2. From $\frac{1}{2}$ subtract $\frac{1}{6}$. *Ans.* $\frac{1}{3}$.
3. From $\frac{1}{2}$ subtract $\frac{1}{8}$. *Ans.* $\frac{3}{8}$.
4. From $\frac{3}{7}$ subtract $\frac{1}{4}$. *Ans.* $\frac{5}{28}$.
5. From $\frac{11}{12}$ subtract $\frac{1}{6}$. *Ans.* $\frac{3}{4}$.
6. From $\frac{1}{18}$ subtract $\frac{1}{360}$. *Ans.* $\frac{19}{360}$.
7. From $\frac{109}{105}$ subtract $\frac{17}{35}$. *Ans.* $\frac{58}{105}$.
8. From $\frac{360}{480}$ subtract $\frac{51}{120}$. *Ans.* $\frac{13}{40}$.

NOTE.—As in addition, if either of the fractions is compound, it must first be reduced to its simplest form.

9. From $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{5}$ subtract $\frac{1}{10}$. *Ans.* $\frac{1}{2}$.
10. From $\frac{6}{7}$ subtract $\frac{5}{9}$ of $\frac{18}{25}$. *Ans.* $\frac{26}{35}$.
11. From $\frac{1}{2}$ of $\frac{3}{7}$ subtract $\frac{1}{7}$ of $\frac{3}{8}$. *Ans.* $\frac{9}{56}$.
12. From $3\frac{1}{5}$ subtract $2\frac{1}{9}$. *Ans.* $1\frac{4}{45}$.
13. From $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}$ subtract $\frac{1}{8}$ of $\frac{2}{7}$. *Ans.* $\frac{37}{140}$.
14. From $\frac{3}{2}$ of $\frac{4}{3}$ of $\frac{5}{4}$ of $\frac{6}{5}$, subtract $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$.
Ans. $2\frac{2}{3}$.

MULTIPLICATION OF FRACTIONS.

79. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

We know (ART. **73**), that $\frac{2}{3}$ multiplied by $\frac{4}{5}$ is the same as $\frac{2}{3}$ of $\frac{4}{5}$. Hence, we must use the same rule as for reducing compound fractions.

Therefore, to multiply fractions, we have this

RULE.

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator: always observing to reject, or cancel, such factors as are common to both numerators and denominators.

If any of the factors are whole numbers, they may be made to take the form of a fraction by giving to them a denominator of 1; (see ART. **30**,) and then the general rule will apply.

What is the Rule for multiplying fractions?

EXAMPLES.

1. Multiply $\frac{1}{2}$ by $\frac{1}{3}$. *Ans.* $\frac{1}{6}$.
2. Multiply $\frac{1}{2}$ by $\frac{1}{4}$. *Ans.* $\frac{1}{8}$.
3. Multiply $\frac{1}{3}$ by $\frac{2}{5}$. *Ans.* $\frac{2}{15}$.
4. Multiply $\frac{1}{3}$ by $\frac{7}{8}$. *Ans.* $\frac{7}{24}$.
5. Multiply $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, all together. *Ans.* $\frac{1}{40}$.
6. Multiply $\frac{4}{5}$ by $\frac{10}{16}$.

In this example, we cancelled the 4 of the numerator against a part of the 16 of the denominator; and 5 of the denominator against a part of the 10 in the numerator. Thus:

$$\frac{\cancel{4}}{\cancel{5}}\times\frac{\overset{2}{\cancel{10}}}{\underset{4}{\cancel{16}}}$$

Finally, cancelling the 2 in the numerator against a part of the 4 in the denominator, we find

$$\frac{\cancel{4}}{\cancel{5}}\times\frac{\overset{\cancel{2}}{\cancel{10}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{16}}}=\frac{1}{2}\,Ans.$$

7. Multiply the fractions $\frac{3}{4}$, $\frac{8}{9}$, $\frac{6}{7}$.

$$\frac{\cancel{3}}{\cancel{4}}\times\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}}\times\frac{\overset{2}{\cancel{6}}}{7}=\frac{4}{7}\,Ans.$$

NOTE.—A little practice will enable the student to perform these operations of cancelling with great ease and rapidity. And since, as was remarked under ART. **73**, it is immaterial which factors are first cancelled, the sim-

plicity of the work must depend much upon his skill and ingenuity.

8. Multiply together the fractions $3\frac{1}{2}$, $4\frac{1}{3}$, $\frac{1}{14}$.

Expressing the multiplication, after reducing them, we have

$$\frac{7}{2}\times\frac{13}{3}\times\frac{1}{14}.$$

Cancelling the 7 of the numerator against a part of the 14 of the denominator, we have

$$\frac{\cancel{7}}{2}\times\frac{13}{3}\times\frac{1}{\underset{2}{\cancel{14}}}=\frac{13}{12}=1\tfrac{1}{12}.$$

9. Multiply together the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. *Ans.* $\frac{1}{5}$.
10. Multiply together the fractions $\frac{3}{7}$, $\frac{4}{5}$, $\frac{5}{9}$, $\frac{14}{3}$. *Ans.* $\frac{8}{9}$.
11. Multiply together the fractions $3\frac{1}{3}$, $4\frac{1}{4}$, $5\frac{1}{5}$. *Ans.* $\frac{221}{3}=73\frac{2}{3}$.
12. Multiply together $\frac{3}{40}$, $\frac{7}{18}$, $\frac{5}{7}$ *Ans.* $\frac{1}{48}$.
13. Multiply $\frac{3}{7}$ by 4. *Ans.* $\frac{12}{7}=1\frac{5}{7}$.
14. Multiply 7 by $\frac{3}{4}$. *Ans.* $\frac{21}{4}=5\frac{1}{4}$.
15. Multiply $7\frac{1}{2}$ by $3\frac{1}{2}$. *Ans.* $\frac{105}{4}=26\frac{1}{4}$.
16. Multiply $16\frac{1}{2}$ by 5. *Ans.* $\frac{165}{2}=82\frac{1}{2}$.

DIVISION OF FRACTIONS.

80. Let us endeavor to divide $\frac{4}{7}$ by $\frac{5}{8}$. We know that $\frac{4}{7}$ can be divided by 5, by multiplying the denominator by 5, (see PROP. II., ART. **68**,) which gives

$$\frac{4}{7\times5}$$

Now, since $\frac{5}{8}$ is but one eighth of 5, it follows that $\frac{4}{7}$ divided by $\frac{5}{8}$ must be eight times as great as $\frac{4}{7}$ divided by 5. Therefore, $\frac{4}{7}$ divided by $\frac{5}{8}$ must be

$$\frac{4\times8}{7\times5}$$

From which we see that $\frac{4}{7}$ has been multiplied by $\frac{5}{8}$ inverted.

Hence, to divide one fraction by another, we have a

RULE.

Reduce the fractions to their simplest form. Invert the divisor, and then proceed as in multiplication.

If either the dividend or divisor is a whole number, it may be converted into an improper fraction having 1 for its denominator.

Repeat the Rule for the division of fractions.

EXAMPLES.

1. Divide $\frac{12}{13}$ by $\frac{4}{26}$.

Inverting the divisor, we have

$$\frac{12}{13}\times\frac{26}{4}.$$

Cancelling, we find

$$\frac{\overset{3}{\cancel{12}}}{\cancel{13}}\times\frac{\overset{2}{\cancel{26}}}{\cancel{4}}=6.$$

2. Divide $\frac{1}{2}$ by $\frac{1}{3}$. *Ans.* $\frac{3}{2}=1\frac{1}{2}$.
3. Divide $\frac{1}{2}$ by $\frac{1}{4}$. *Ans.* 2.
4. Divide $\frac{3}{4}$ by $\frac{1}{3}$. *Ans.* $\frac{9}{4}=2\frac{1}{4}$.
5. Divide $\frac{6}{7}$ by $\frac{6}{11}$. *Ans.* $\frac{11}{7}=1\frac{4}{7}$.
6. Divide $\frac{8}{15}$ by $\frac{12}{20}$. *Ans.* $\frac{8}{9}$.
7. What is the quotient of $4\frac{1}{3}$ divided by $17\frac{1}{2}$? *Ans.* $\frac{26}{105}$.
8. What is the quotient of $\frac{15}{17}$ divided by 10? *Ans.* $\frac{3}{34}$.
9. Divide $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{1}{6}$ of $\frac{5}{8}$. *Ans.* $\frac{18}{5}=3\frac{3}{5}$.
10. Divide $3\frac{1}{3}$ of $2\frac{1}{2}$ by $4\frac{1}{4}$. *Ans.* $\frac{100}{51}=1\frac{49}{51}$.
11. Divide $\frac{1}{2}$ by $\frac{3}{8}$ of $\frac{4}{5}$. *Ans.* $\frac{5}{3}=1\frac{2}{3}$.

RECIPROCALS OF NUMBERS.

81. The *reciprocal* of a number is the result obtained by dividing 1 by the number. Thus, the reciprocals of 2, 3, 4, and 5, are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$. From this we discover that the reciprocal of an *integer*, or whole number, is equal to a vulgar fraction whose numerator is 1, and whose denominator is the given number.

The reciprocal of $\frac{3}{5}$ is found by dividing 1 by $\frac{3}{5}$, which (Art. **80**), is $1 \div \frac{3}{5} = 1 \times \frac{5}{3} = \frac{5}{3}$.

In the same way we find the reciprocal of $\frac{7}{9}$ to be $\frac{9}{7}$, and in general, the reciprocal of a vulgar fraction is the value of the fraction when *inverted*.

EXAMPLES.

1. What are the reciprocals of 7, 8, 9, 10, 11?
Ans.. $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$.

2. What are the reciprocals of 18, 23, and 41?
Ans. $\frac{1}{18}$, $\frac{1}{23}$, $\frac{1}{41}$.

3. What are the reciprocals of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$?
Ans. $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$.

4. What are the reciprocals of $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$?
Ans. $\frac{2}{3}$, $\frac{3}{7}$, $\frac{4}{13}$.

5. What are the reciprocals of $\frac{3}{4}$ of $\frac{3}{7}$, $\frac{2}{3}$ of $\frac{7}{5}$?
Ans. $\frac{4}{3}$ of $\frac{7}{3}$, $\frac{3}{2}$ of $\frac{5}{7}$.

DENOMINATE FRACTIONS.

82. Undor Art. **46**, we defined a denominate number, or integer, as one whose unit was of a particular denomination or kind. For a similar reason, a *denominate fraction* is a part of a particular kind of unit. Thus, $\frac{1}{2}$ of a yard is a denominate fraction, expressing a part of the particular unit one yard; $\frac{2}{3}$ of a pound is also a denominate fraction, expressing a part of the particular unit one pound.

We know (by Art. **62**), that denominate numbers may be changed or reduced from one denomination to another without altering their values. By a similar method may denominate fractions be reduced from one name to another.

What have we already defined a denominate number to be? What is a denominate fraction? Give some examples. May denominate fractions be changed from one name to another without altering their values?

REDUCTION OF DENOMINATE FRACTIONS.

83. Suppose we wish to reduce $\frac{1}{360}$ of a pound sterling to an equivalent fraction of a farthing, we proceed as

follows: since there are 20 shillings in a pound, it follows that $\frac{1}{360}$ of a pound is the same as 20 times $\frac{1}{360}$ of a shilling, and this is also the same as 12 times 20 times $\frac{1}{360}$ of a penny; which, in turn, is 4 times 12 times 20 times $\frac{1}{360}$ of a farthing. Hence, $\frac{1}{360}$ of a pound sterling is equivalent to $\frac{1}{360}$ of $\frac{20}{1}$ of $\frac{12}{1}$ of $\frac{4}{1}$ of a farthing.

Again, let us reduce $\frac{3}{5}$ of a farthing to an equivalent fraction of a pound sterling. In this case we must use the reciprocals of $\frac{20}{1}$, $\frac{12}{1}$, $\frac{4}{1}$, we thus find that $\frac{3}{5}$ of a farthing is equivalent to $\frac{3}{5}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound sterling.

Hence, to reduce fractions of one denominate value to equivalent fractions of other denominate values, we have this

RULE.

I. When the given fraction is to be reduced to a higher denomination, multiply it by a compound fraction whose terms are the reciprocals of the successive denominate values, included between the denomination of the given fraction, and the one to which it is to be reduced.

II. When the given fraction is to be reduced to a lower denomination, then multiply it by a compound fraction, whose terms have units for their denominators, and for numerators the successive denominate values included between the denomination of the given fraction, and the one to which it is to be reduced.

EXAMPLES.

1. Reduce $\frac{3}{8}$ of an inch to the fraction of a mile.

In this example, the different denominate values between an inch and a mile are 12 inches, $16\frac{1}{2}=\frac{33}{2}$ feet, 40 rods, and 8 furlongs; our compound fraction is $\frac{1}{12}$ of $\frac{2}{33}$ of $\frac{1}{40}$ of $\frac{1}{8}$, which multiplied by the given fraction produces $\frac{3}{8}$ of $\frac{1}{12}$ of $\frac{2}{33}$ of $\frac{1}{40}$ of $\frac{1}{8}$; cancelling the 3 and 2 of the numerators against a part of 12 of the denominator, we get

$$\frac{\cancel{3}}{8}\times\frac{1}{\underset{2}{\cancel{12}}}\times\frac{\cancel{2}}{33}\times\frac{1}{40}\times\frac{1}{8}=\frac{1}{168960}.$$

Therefore, $\frac{3}{8}$ of an inch is equivalent to $\frac{1}{168960}$ of a mile.

2. Reduce $\frac{3}{11520}$ of a solar day to the fraction of a second.

In this example, the successive denominate values between a solar day and a second, are 24 hours, 60 minutes, and 60 seconds; hence, the compound fraction will be $\frac{24}{1}$ of $\frac{60}{1}$ of $\frac{60}{1}$, which, multiplied by the given fraction becomes

$$\frac{3}{11520}\times\frac{24}{1}\times\frac{60}{1}\times\frac{60}{1}.$$

Cancelling 60 of the numerator against a part of the denominator 11520, we have

$$\frac{3}{\begin{array}{c}\cancel{11520}\\ 192\end{array}}\times\frac{24}{1}\times\frac{\cancel{60}}{1}\times\frac{60}{1}$$

Again, cancelling the 24 of the numerator against a part of the denominator 192, we get

$$\frac{3}{\begin{array}{c}\cancel{11520}\\ \cancel{192}\\ 8\end{array}}\times\frac{\cancel{24}}{1}\times\frac{\cancel{60}}{1}\times\frac{60}{1}$$

Finally, cancelling the factor 4, which is common to the numerator 60, and the denominator 8, we have

$$\frac{3}{\begin{array}{c}\cancel{11520}\\ \cancel{192}\\ \cancel{8}\\ 2\end{array}}\times\frac{\cancel{24}}{1}\times\frac{\cancel{60}}{1}\times\frac{\begin{array}{c}15\\ \cancel{60}\end{array}}{1}=\frac{45}{2}\text{ of a second.}$$

We have been particular to give the complete work of cancelling in these examples, by writing down the whole work at the successive stages of operation. In practice, the expression need not be written more than once; with a little practice the student will be able to strike out the common factors with accuracy and despatch.

3. Reduce $\frac{1}{4032}$ of a pipe of wine to an equivalent fraction of a gill.

In this example, the successive denominate values between a pipe and a gill are 2 hogsheads, 63 gallons, 4 quarts, 2 pints and 4 gills; therefore our compound fraction is $\frac{2}{1}$ of $\frac{63}{1}$ of $\frac{4}{1}$ of $\frac{2}{1}$ of $\frac{4}{1}$, which, multiplied by the given fraction, becomes $\frac{1}{4032}$ of $\frac{2}{1}$ of $\frac{63}{1}$ of $\frac{4}{1}$ of $\frac{2}{1}$ of $\frac{4}{1}$; this becomes, after cancelling like factors, 1 gill.

4. Reduce $\frac{144}{360}$ of a yard to the fraction of a mile.
Ans. $\frac{1}{4400}$.

5. Reduce $\frac{27}{84}$ of a gill to the fraction of a gallon.
Ans. $\frac{9}{896}$.

6. Reduce $\frac{330}{495}$ of a pound to the fraction of a ton.
Ans. $\frac{1}{3360}$.

7. Reduce $\frac{1}{3}$ of a mile to the fraction of a foot.
Ans. 1760 feet.

8. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{2}{7}$ of a yard to the fraction of a mile.
Ans. $\frac{1}{36960}$.

9. Reduce $\frac{1}{6}$ of $\frac{1}{8}$ of $\frac{24}{28}$ of a gallon to the fraction of a gill.
Ans. $\frac{4}{7}$.

10. Reduce $\frac{2}{3}$ of $\frac{4}{9}$ of a hogshead of wine to the fraction of a gill.
Ans. $\frac{1792}{3}=597\frac{1}{3}$.

11. Reduce $\frac{1}{2}$ of $\frac{3}{7}$ of $4\frac{1}{2}$ yards to the fraction of an inch.
Ans. $\frac{243}{7}=34\frac{5}{7}$.

12. Reduce $\frac{1}{7}$ of $\frac{3}{43}$ of a farthing to the fraction of a shilling.
Ans. $\frac{1}{4816}$.

13. Reduce $\frac{7}{59}$ of an ounce to the fraction of a pound avoirdupois.
Ans. $\frac{7}{944}$.

84. To find what fractional part one quantity is of another of the same kind, but of different denominations.

Suppose we wish to know what part of 1 yard 2 feet 3 inches is; we reduce 1 yard to inches, which gives 1 yard $=36$ inches; we also reduce 2 feet 3 inches to inches, which gives 2 feet 3 inches $=27$ inches. Now it is obvious that 2 feet 3 inches is the same part of a yard that 27 is of 36, which is $\frac{27}{36}=\frac{3}{4}$.

Hence, we deduce this

RULE.

Reduce the given quantities to the lowest denomination mentioned in either, then divide the number which is to become the fractional part, by the other number.

EXAMPLES.

1. What part of £3 4*s*. 1*d*. is 2*s*. 6*d*.?

In this example, the quantities when reduced become £3 4*s*. 1*d*.=769*d*.; and 2*s*. 6*d*.=30*d*.; therefore, $\frac{30}{769}$ is the fractional part which 2*s*. 6*d*. is of £3 4*s*. 1*d*.

2. What part of 3 miles 40 rods is 27 feet 9 inches? *Ans*. $\frac{37}{22000}$.

3. What part of a day is 17 minutes 4 seconds? *Ans*. $\frac{8}{675}$.

4. What part of \$700 is \$5.30? *Ans*. $\frac{53}{7000}$.

5. What fractional part of 2 hogsheads is 3 pints? *Ans*. $\frac{1}{336}$.

6. What part of \$3 is $2\frac{1}{2}$ cents? *Ans*. $\frac{1}{120}$.

7. What part of 10 shillings 8 pence is 3 shillings 1 penny? *Ans*. $\frac{37}{128}$.

8. What part of 100 acres is 63 acres 2 roods 7 rods of land? *Ans*. $\frac{10167}{16000}$.

M

85. To reduce a fraction of any given denomination to whole denominate numbers.

Suppose we wish to know the value of $\frac{3}{8}$ of a yard; we know that $\frac{3}{8}$ of a yard equals $\frac{3}{8}$ of $\frac{4}{1}$ of a quarter$=\frac{3}{2}$ of a quarter$=1$ quarter$+\frac{1}{2}$ of a quarter.

Again, $\frac{1}{2}$ of a quarter equals $\frac{1}{2}$ of $\frac{4}{1}$ of a nail$=2$ nails. Therefore, $\frac{3}{8}$ of a yard equals 1 quarter and 2 nails.

Hence, we deduce this

RULE.

Multiply the numerator by the units in the next inferior denominate value, and divide the product by the denominator; multiply the remainder, if any, by the next lower denominate value, and again divide the product by the denominator; con-

tinue this process until there is no remainder, or until we reach the lowest denominate value. The successive quotients will form the successive denominate values.

EXAMPLES.

1. What is the value of $\frac{3}{15}$ of an hour?

In this example, $\frac{3}{15}$ of an hour equal $\frac{3}{15}$ of $\frac{60}{1}$ of a minute, equals 12 minutes.

2. What is the value of $\frac{3}{7}$ of 1 yard.

Ans. 1 quarter $2\frac{6}{7}$ nails.

3. What is the value of $\frac{1}{3}$ of $\frac{3}{8}$ of 1 mile?

Ans. 1 furlong 20 rods.

4. What is the value of $\frac{3}{7}$ of $\frac{5}{6}$ of 1 cwt.?

Ans. 1 quarter 12 pounds.

5. What is the value of $\frac{1}{6}$ of 14 miles 6 furlongs?

Ans. 2 miles 3 furlongs 26 rods 11 feet.

6. What is the value of $\frac{1}{3}$ of $\frac{3}{5}$ of 2 days of 24 hours each? *Ans.* 9 hours 36 minutes.

7. What is the value of $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{3}{14}$ of an hour?

Ans. 5 minutes $37\frac{1}{2}$ seconds.

ADDITION OF DENOMINATE FRACTIONS.

86. So long as fractions are of different denominate values, they cannot be added, any more than integers can of different denominations. Hence, they must first be reduced to the same denomination; then we must reduce them to a common denominator, and apply the rule under ART. **77**.

What is the rule for the division of denominate fractions?

EXAMPLES.

1. Add $\frac{1}{5}$ of a shilling to $\frac{1}{4}$ of a pound.

I. $\frac{1}{5}$ of a shilling equals $\frac{1}{5}$ of $\frac{1}{20}$ of a pound$=\frac{1}{100}$ of a pound, which added to $\frac{1}{4}$ of a pound$=\frac{25}{100}$ of a pound, gives $\frac{26}{100}=\frac{13}{50}$ of a pound, for the sum.

II. $\frac{1}{4}$ of a pound$=\frac{1}{4}$ of $\frac{20}{1}$ of a shilling$=5$ shillings,

which added to $\frac{1}{5}$ of a shilling, gives $5\frac{1}{5}=\frac{26}{5}$ of a shilling for the sum.

If our work is right, these two results ought to be of the same value, that is, $\frac{13}{50}$ of a pound must equal $5\frac{1}{5}$ shillings.

We know that $\frac{13}{50}$ of a pound $=\frac{13}{50}$ of $\frac{20}{1}$ of a shilling $=\frac{26}{5}$ of a shilling.

2. Add $\frac{1}{3}$ of a yard, $\frac{5}{6}$ of a foot, and $\frac{3}{8}$ of a mile.

These fractions, before adding, might be reduced to fractions of a yard, or of a foot, or of a mile, or of any of the denominate values of Long Measure. But a better way will be to reduce each to its integral denominate value, by Rule under ART. **85**.

Thus: $\frac{1}{3}$ of a yard $=\frac{1}{3}$ of $\frac{3}{1}$ of a foot $=1$ foot.
$\frac{5}{6}$ of a foot $=\frac{5}{6}$ of $\frac{12}{1}$ of an inch $=10$ inches.
$\frac{3}{8}$ of a mile $=\frac{3}{8}$ of $\frac{8}{1}$ of a furlong $=3$ furlongs.
Therefore, the sum is 3 furlongs, 1 foot, 10 inches.

3. Add $\frac{1}{2}$ of a week, $\frac{1}{6}$ of a day, $\frac{1}{4}$ of an hour.

$\frac{1}{2}$ of a week $=\frac{1}{2}$ of $\frac{7}{1}$ of a day $=3\frac{1}{2}$ days $=3$ days $+\frac{1}{2}$ of $\frac{24}{1}$ of an hour $=3$ days 12 hours.
$\frac{1}{6}$ of a day $=\frac{1}{6}$ of $\frac{24}{1}$ hour $=4$ hours.
$\frac{1}{4}$ of an hour $=\frac{1}{4}$ of $\frac{60}{1}$ of a minute $=15$ minutes.
Hence, the sum is 3 days, 16 hours, 15 minutes.

4. Add $\frac{1}{5}$ of a year, $\frac{2}{7}$ of a week, $\frac{1}{12}$ of a day, together.
Ans. 75*da.* 2*hr.*

5. What is the sum of $\frac{1}{2}$ of a *cwt.*, $\frac{1}{3}$ of a *qr.*, $\frac{1}{4}$ of a *lb.*?
Ans. 2*qr.* 9*lb.* 9*oz.* $5\frac{1}{3}$*dr.*

6. What is the sum of $\frac{1}{10}$ of a bushel, $\frac{1}{6}$ of a peck, $\frac{1}{2}$ of a quart? *Ans.* $5\frac{1}{30}$*qt.*

7. What is the sum of $\frac{1}{10}$ of a yard, and $\frac{1}{3}$ of a foot?
Ans. $7\frac{3}{5}$ inches.

8. What is the sum of $\frac{2}{3}$ of a week, $\frac{1}{5}$ of a day, and $\frac{1}{3}$ of an hour? *Ans.* 4*da.* 21*hr.* 8*m.*

9. What is the sum of $\frac{2}{3}$ of a bushel, $\frac{1}{3}$ of a peck, and $\frac{1}{7}$ of a quart? *Ans.* 3*pk.* 0*qt.* $0\frac{2}{7}$*pt.*

SUBTRACTION OF DENOMINATE FRACTIONS.

87. As in addition, the fractions must be first reduced to like denominations, afterwards they must be brought to a common denominator, and then the work may be completed, by Rule under ART. 78.

What is the Rule for the subtraction of denominate fractions?

EXAMPLES.

1. From $\frac{1}{8}$ of a pound subtract $\frac{1}{5}$ of a shilling.

I. $\frac{1}{8}$ of a £ $=\frac{1}{8}$ of $\frac{20}{1}$ of a shilling $=\frac{5}{2}$ of a shilling.

Therefore, $\frac{5}{2}-\frac{1}{5}=\frac{25}{10}-\frac{2}{10}=\frac{23}{10}$. So that the difference is $\frac{23}{10}$ of a shilling $=2\frac{3}{10}$ of a shilling.

II. $\frac{1}{5}$ of a shilling $=\frac{1}{5}$ of $\frac{1}{20}$ of a pound $=\frac{1}{100}$ of a pound.

And $\frac{1}{8}-\frac{1}{100}=\frac{25}{200}-\frac{2}{200}=\frac{23}{200}$. So that the difference is $\frac{23}{200}$ of a pound $=\frac{23}{200}$ of $\frac{20}{1}$ of a shilling $=\frac{23}{10}$ of a shilling, as before.

2. From $\frac{3}{8}$ of a day subtract $\frac{1}{5}$ of a minute.

$\frac{3}{8}$ of a day $=\frac{3}{8}$ of $\frac{24}{1}$ of an hour $=9$ hours.

$\frac{1}{5}$ of a minute $=\frac{1}{5}$ of $\frac{60}{1}$ of a second $=12$ seconds.

		hr.	*m.*	*sec.*
Hence,	From	9	0	0
	Take	0	0	12
	Difference	8	59	48

3. From $\frac{1}{2}$ of $\frac{2}{3}$ of 15 yards of cloth, subtract $\frac{1}{8}$ of $\frac{8}{13}$ of one quarter.

$\frac{1}{2}$ of $\frac{2}{3}$ of 15 yards $=5$ yards.

$\frac{1}{8}$ of $\frac{8}{13}$ of one quarter $=\frac{1}{8}$ of $\frac{8}{13}$ of $\frac{4}{1}$ of a nail $=\frac{4}{13}$ of a nail.

		yd.	*qr.*	*na.*
Hence,	From	5	0	0
	Take	0	0	$0\frac{4}{13}$
	Difference	4	3	$3\frac{9}{13}$

4. From $\frac{1}{7}$ of 5 acres of land, subtract $\frac{1}{4}$ of 3 roods.

Ans. 2*R.* $4\frac{2}{7}$*P.*

5. From $\frac{2}{5}$ of an ounce, take $\frac{3}{8}$ of a pennyweight.

Ans. 7*pwt.* 15*gr.*

6 From $\frac{1}{9}$ of a hogshead, take $\frac{2}{3}$ of a quart.

Ans. 6*gal.* 3*qt.* $\frac{2}{3}$*pt*

88. EXERCISES IN VULGAR FRACTIONS.

1. Reduce $\frac{24}{48}$ to its lowest terms. *Ans.* $\frac{1}{2}$.
2. Reduce $\frac{8}{64}$ to its lowest terms. *Ans.* $\frac{1}{8}$.
3. Reduce $\frac{14}{16}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
4. Reduce $\frac{588}{672}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
5. Reduce $\frac{3357}{4476}$ to its lowest terms. *Ans.* $\frac{3}{4}$.
6. Reduce $\frac{7721}{7763}$ to its lowest terms. *Ans.* $\frac{1103}{1109}$.
7. Reduce $\frac{15636}{16939}$ to its lowest terms. *Ans.* $\frac{12}{13}$.
8. Reduce $\frac{505}{515}$ to its lowest terms. *Ans.* $\frac{101}{103}$.
9. Reduce $\frac{18909}{21010}$ to its lowest terms. *Ans.* $\frac{9}{10}$.
10. Reduce $\frac{515}{505}$ to a mixed number. *Ans.* $1\frac{2}{101}$.
11. Reduce $\frac{37}{5}$ to a mixed number. *Ans.* $7\frac{2}{5}$.
12. Reduce $\frac{48}{6}$ to a whole number. *Ans.* 8.
13. Reduce $\frac{59}{17}$ to a mixed number. *Ans.* $3\frac{8}{17}$.
14. Reduce $\frac{2371}{907}$ to a mixed number. *Ans.* $2\frac{557}{907}$.
15. Reduce $\frac{101}{97}$ to a mixed number. *Ans.* $1\frac{4}{97}$.
16. Reduce $3\frac{1}{2}$ to an improper fraction. *Ans.* $\frac{7}{2}$.
17. Reduce $15\frac{11}{13}$ to an improper fraction. *Ans.* $\frac{206}{13}$.
18. Reduce $3\frac{7}{17}$ to an improper fraction. *Ans.* $\frac{58}{17}$.
19. Reduce $1\frac{9}{101}$ to an improper fraction. *Ans.* $\frac{110}{101}$.
20. Reduce $100\frac{11}{17}$ to an improper fraction. *Ans.* $\frac{1711}{17}$.
21. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to its simplest form. *Ans.* $\frac{1}{4}$.
22. Reduce $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{20}{27}$ to its simplest form. *Ans.* $\frac{1}{3}$.
23. Reduce $\frac{7}{8}$ of $\frac{1}{2}$ of $\frac{6}{14}$ of 3 to its simplest form. *Ans.* $\frac{9}{16}$.
24. Reduce $\frac{1}{10}$ of $\frac{3}{7}$ of $\frac{21}{25}$ of $3\frac{1}{3}$ to its simplest form. *Ans.* $\frac{3}{25}$.
25. Reduce $\frac{6}{7}$ of $\frac{14}{3}$ of $\frac{1}{2}$ of 100 to its simplest form. *Ans.* 200.
26. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, to equivalent fractions having a common denominator. *Ans.* $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$.
27. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, to equivalent fractions having a common denominator. *Ans.* $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, $\frac{10}{60}$.
28. Reduce $3\frac{1}{2}$, $\frac{7}{5}$, $\frac{3}{4}$, $\frac{3}{10}$, to equivalent fractions having a common denominator. *Ans.* $\frac{70}{20}$, $\frac{28}{20}$, $\frac{15}{20}$, $\frac{6}{20}$.
29. Reduce $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{11}$, to equivalent fractions having a common denominator. *Ans.* $\frac{385}{1155}$, $\frac{231}{1155}$, $\frac{165}{1155}$, $\frac{105}{1155}$.
30. Reduce $\frac{3}{5}$, $\frac{5}{7}$, $\frac{7}{11}$, $\frac{11}{13}$, to equivalent fractions having a common denominator. *Ans.* $\frac{3003}{5005}$, $\frac{3575}{5005}$, $\frac{3185}{5005}$, $\frac{4235}{5005}$.

31. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$? *Ans.* $\frac{13}{12}=1\frac{1}{12}$.

32. What is the sum of $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$? *Ans.* $\frac{143}{60}=2\frac{23}{60}$.

33. From a piece of cloth $\frac{1}{2}$ and $\frac{1}{3}$ of the whole was cut off. What part of the whole was thus taken away? *Ans.* $\frac{5}{6}$.

34. From $\frac{1}{2}$ subtract $\frac{1}{3}$. *Ans.* $\frac{1}{6}$.

35. From $\frac{1}{10}$ subtract $\frac{1}{11}$. *Ans.* $\frac{1}{110}$.

36. From $\frac{3}{4}$ subtract $\frac{4}{7}$. *Ans.* $\frac{5}{28}$.

37. A tree 150 feet high had $\frac{1}{5}$ broken off in a storm. What was the length broken off? *Ans.* 30 feet.

38. A and B together possess 1477 sheep, of which A owns $\frac{4}{7}$ and B $\frac{3}{7}$. How many belong to each man?

Ans. { A's 844. B's 633.

39. A owns $\frac{3}{11}$ of a ship, valued at $15422; he sells to B $\frac{2}{3}$ of his share. What is the value of what A has left; also, what is the value of B's part?

Ans. { A's remaining part is $1402. B's part is $2804.

40. A cotton mill is sold for $30000, of which A owns $\frac{1}{5}$ of the whole, B and C each own $\frac{1}{2}$ of $\frac{1}{3}$ of the whole. How many dollars does each one claim?

Ans. { A claims $6000. B claims $5000. C claims $5000.

41. A and B have a melon, of which A owns $\frac{3}{8}$ and B $\frac{5}{8}$; C offers them one shilling, to partake equally with them of the melon, which was agreed to. How must the shilling be divided between A and B?

Ans. { A must have $\frac{1}{8}$ of it. B must have $\frac{7}{8}$ of it.

42. A farmer had $\frac{1}{5}$ of his sheep in one field, $\frac{1}{6}$ in a second field, and the residue, which was 779, in a third field? How many sheep had he in all? *Ans.* 1230.

43. A person gave $\frac{1}{7}$ of a pound for a hat, $\frac{1}{3}$ of a shilling for some thread, and $\frac{1}{2}$ of a penny for a needle. What did he pay for all? *Ans.* 3*s*. 2*d*. $3\frac{1}{7}$*far*.

44. What is the value of $\frac{1}{2}$ of a week, $\frac{1}{3}$ of a day, and $\frac{1}{4}$ of a minute? *Ans.* 3*da*. 20*hr*. 15*sec*.

45. What is the value of $\frac{1}{5}$ of a pound, $\frac{1}{4}$ of an ounce, and $\frac{1}{7}$ of a pennyweight Troy? *Ans.* 2*oz.* 13*pwt.* $3\frac{3}{7}$*gr.*

46. If $4\frac{1}{3}$ pounds of sugar cost $43\frac{1}{3}$ cents, how much is it per pound? *Ans.* 10 cents.

47. If I pay $4.04 for $8\frac{3}{4}$ bushels of apples, how much do I give per bushel? *Ans.* $46\frac{6}{35}$ cents.

48. Four persons, A, B, C, and D, own a ship, of which A owns $\frac{1}{5}$ of $\frac{2}{3}$ of the whole; B owns $\frac{7}{8}$ of $\frac{2}{3}$ as much as A; C owns $\frac{3}{4}$ as much as B; and D owns the remainder. What are the respective parts owned by each?

$$\textit{Ans.}\begin{cases} \text{A owned } \frac{48}{360}. \\ \text{B } \quad \text{“} \quad \frac{28}{360}. \\ \text{C } \quad \text{“} \quad \frac{21}{360}. \\ \text{D } \quad \text{“} \quad \frac{263}{360}. \end{cases}$$

49. A certain sum of money is to be divided between 4 persons, in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is $28. What is the sum?

$\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{3}{4}$, which wants just $\frac{1}{4}$ of being the whole; hence, the fourth one had $\frac{1}{4}$ of the whole. Consequently, \$28 is $\frac{1}{4}$ of the whole, and the whole is $\$28\times4=\$112$.

50. A received $\frac{1}{6}$ of a legacy, B $\frac{1}{10}$, and C the remainder. Now it is found that A had $80 more than B. How much did each receive?

$\frac{1}{6}-\frac{1}{10}=\frac{1}{15}$. Hence, \$80 was $\frac{1}{15}$ of the whole legacy; the legacy was therefore $\$80\times15=\$1200$.

Hence,

A had $\frac{1}{6}$ of \$1200	=\$200
B had $\frac{1}{10}$ of \$1200	=\$120
C had the remainder	=\$880
Proof	\$1200

VULGAR FRACTIONS REDUCED TO DECIMALS.

89. To change a vulgar fraction into an equivalent decimal fraction.

It is obvious that the rule under Art. **85**, will apply to

this case, by considering all the denominate values as decreasing regularly in a tenfold ratio. Hence, this

RULE.

Annex a cipher to the numerator, and then divide by the denominator; to the remainder annex another cipher, and again divide by the denominator; and so continue to do until there is no remainder, or until we have obtained as many decimal figures as may be desired. The successive quotients will be the successive decimal figures required.

EXAMPLES.

1. What decimal fraction is equivalent to $\frac{1}{16}$?

$$
\begin{array}{r}
16)100(0.0625 \\
\underline{96} \\
40 \\
\underline{32} \\
80 \\
\underline{80} \\
0
\end{array}
$$

2. What decimal is equivalent to $\frac{1}{18}$? *Ans.* 0.05555, &c.
3. What decimal is equivalent to $\frac{1}{20}$? *Ans.* 0.05.
4. What decimal is equivalent to $\frac{1}{25}$? *Ans.* 0.04.
5. What decimal is equivalent to $\frac{1}{3}$? *Ans.* 0.3333, &c.

When the decimal figures obtained by converting a vulgar fraction into decimals do not terminate, they must recur in periods, whose number of terms cannot exceed the number of units in the denominator, less one. For all the different remainders which occur must be less than the denominator; and therefore their number can not exceed the denominator, less one; and whenever we obtain a remainder like one that has previously occurred, then the decimal figures will begin to repeat. Decimals which recur in this way, are called *repetends*.

When the period begins with the first decimal figure, it is called a *simple repetend.* But when other decimal figures occur before the period commences, it is called a *compound repetend.*

A repetend is distinguished from ordinary decimals by a period or dot placed over the first and last figure of the circulating period.

90. The following vulgar fractions give simple repetends:

$$\tfrac{1}{3}=0.\dot{3}.$$
$$\tfrac{1}{7}=0.\dot{1}4285\dot{7}.$$
$$\tfrac{1}{9}=0.\dot{1}.$$
$$\tfrac{1}{11}=0.\dot{0}\dot{9}.$$
$$\tfrac{1}{13}=0.\dot{0}7692\dot{3}.$$
$$\tfrac{1}{17}=0.\dot{0}58823529411764\dot{7}.$$
$$\tfrac{1}{19}=0.\dot{0}5263157894736842\dot{1}.$$
$$\tfrac{1}{21}=0.\dot{0}4761\dot{9}.$$
$$\tfrac{1}{23}=0.\dot{0}43478260869565217391\dot{3}.$$

91. The following ones give compound repetends:

$$\tfrac{1}{6}=0.1\dot{6}.$$
$$\tfrac{1}{12}=0.08\dot{3}.$$
$$\tfrac{1}{14}=0.0\dot{7}1428\dot{5}.$$
$$\tfrac{1}{15}=0.0\dot{6}.$$
$$\tfrac{1}{18}=0.0\dot{5}.$$
$$\tfrac{1}{22}=0.0\dot{4}\dot{5}.$$
$$\tfrac{1}{24}=0.041\dot{6}.$$

92. Those simple repetends, which have as many terms, less one, as there are units in the denominator, we shall call *perfect repetends.* The following are some of the perfect repetends:

$\frac{1}{7}=0.\dot{1}4285\dot{7}.$

$\frac{1}{17}=0.\dot{0}58823529411764\dot{7}.$

$\frac{1}{19}=0.\dot{0}5263157894736842\dot{1}.$

$\frac{1}{23}=0.\dot{0}43478260869565217391\dot{3}.$

$\frac{1}{29}=0.\dot{0}34482758620689655172413793\dot{1}.$

NOTE.—For some interesting properties of *repetends*, see Higher Arithmetic.

REDUCTION OF DENOMINATE DECIMALS.

93. A *denominate decimal* is a decimal fraction of a unit of a particular kind. Thus, 0.45 of a £, is a denominate decimal, since the unit is £1; for the same reason, 0.25 of a foot is a denominate decimal, the unit being 1 foot.

What is a denominate decimal? Give some examples.

CASE I.

To reduce denominate numbers of different denominations to a decimal of a given denomination.

Let it be required to reduce 15*s*. 6*d*. 3*far*. to the decimal of a £.

I. 3*far*.$=\frac{3}{4}$*d*.$=0.75$*d*.

II. 6*d*. 3*far*. is therefore the same as 6.75*d*.; if we divide this by 12, it will become

$$\frac{6.75}{12}=0.5625s.$$

III. 15*s*. 6*d*. 3*far*.$=15.5625$*s*.; this divided by 20, gives

$$\frac{15.5625}{20}=0.778125 \text{ of a £}$$

for the decimal sought. The work may be more concisely done, as in the following

OPERATION.

$$\begin{array}{r|l} 4 & \underline{3 far.} \\ 12 & \underline{6.75d.} \\ 2|0 & \underline{15.5625s.} \\ & \underline{0.778125} \text{ of a £} \end{array}$$

EXPLANATION.

We placed the different denominations above each other, so that the smallest denomination stood at the top; we then divided the 3 farthings by 4, since 4 farthings make one penny, and the quotient, which must be a decimal, we placed at the right of the 6*d.*; we next divided 6.75*d.* by 12, because 12 pence make one shilling, and the quotient, which is also a decimal, we placed at the right of the 15*s.*; finally, we divided the 15.5625*s.* by 20, because 20 shillings make one pound. In dividing by 20, we cut off the cipher, and then divided by 2, observing to remove the decimal point one place to the left.

We therefore have this

RULE.

Place the different denominations above each other, so that the lowest may stand at the top; commencing at the top, divide each denomination by its value in the next denomination; the last quotient will be the decimal required.

Repeat this rule.

EXAMPLES.

1. Reduce £8 5*s.* 2*d.* 1*qr.* to the decimal of a £.

OPERATION.

$$\begin{array}{r|l} 4 & \underline{1} \\ 12 & \underline{2.25} \\ 2|0 & \underline{5.1875} \\ & \underline{8.259375} \text{ of a £.} \end{array}$$

2 Reduce 3*qr.* 2*na.* to the decimal of a yard.

OPERATION.

$$\begin{array}{r|l} 4 & 2 \\ 4 & 3.5 \\ & 0.875 \text{ of a yard.} \end{array}$$

3. Reduce 1*ft.* 4*in.* to the decimal of a yard

OPERATION.

$$\begin{array}{r|l} 12 & 4 \\ 3 & 1.3333 \text{ \&c.} \\ & 0.4444 \text{ \&c. of a yard.} \end{array}$$

4. Reduce 3*lb.* 4*oz.* 8*pwt.* 1*gr.* Troy, to the decimal of a pound. *Ans.* 3.36684027777, &c. of a *lb.*

5. Reduce 3*h.* 30*m.* 10*sec.* to the decimal of a day. *Ans.* 0.145949074074, &c., of a day.

6. Reduce £3 5*s.* 0*d.* 2*far.* to the value of a £. *Ans.* £3.252083333, &c.

7. Reduce 28 gallons of wine to the decimal of a hogshead. *Ans.* 0.4444, &c. of a hogshead.

8. Reduce 4*s.* 6½*d.* to the decimal of a £. *Ans.* £0.2270833, &c.

9. Reduce 18*s.* 3¾*d.* to the decimal of a £. *Ans.* £0.915625.

10. Reduce 3 pecks 5 quarts and 1 pint to the decimal of a bushel. *Ans.* 0.921875 of a bushel.

11. Reduce 11*hr.* 16*m.* 15*sec.* to the decimal of a day. *Ans.* 0.469618055, &c., of a day.

12. Reduce 20 rods 4 yards 2 feet and 6 inches to the decimal of a furlong. *Ans.* 0.521969696, &c. of a furlong.

CASE II.

Find the value of 0.778125 of a £.

OPERATION.

0.778125	of a £.
20	shillings in £1.
15.562500	of a shilling.
12	pence in 1*s*.
11250	
5625	
6.7500	of a penny.
4	farthings in 1 penny.
3.00	farthings.

Which gives 15*s*. 6*d*. 3*far*.

EXPLANATION.

We first multiplied the decimal of a £ by 20, because 20 shillings make 1 pound; pointing off by the rule for decimals, we found 15*s*. and 0.5625 of a shilling. Then we multiplied this decimal of a shilling by 12, because 12 pence make 1 shilling; pointing off, we found 6*d*. and 0.75 of a penny, which being multiplied by 4, because 4 farthings make 1 penny, gave just 3 farthings.

By carefully considering the above operation, we deduce this

RULE.

Multiply the decimal by the number expressing the next lower denomination; point off by the usual rule for decimals; multiply the decimal part, thus pointed off, by the number expressing the next inferior denomination; and so continue to the lowest denomination; the several denominate values sought will appear at the left of the decimal point of the successive products.

Repeat this Rule.

EXAMPLES.

1. What is the value of 0.9075 of an acre?

OPERATION.

$$\begin{array}{r} 0.9075 \\ 4 \text{ roods} = 1A. \\ \hline R.\ 3.6300 \\ 40 \text{ rods} = 1R. \\ \hline P.\ 25.2 \\ \hline \end{array}$$

Ans. 3*R*. 25.2*P*.

2. What is the value of £0.125? *Ans.* 2*s*. 6*d*.
3. What is the value of £0.66$\frac{2}{3}$? *Ans.* 13*s*. 4*d*.
4. What is the value of 0.375 of a hogshead of wine? *Ans.* 23*gal*. 2*qt*. 1*pt*.
5. What is the value of 0.121212 of a year of 365 days? *Ans.* 44*da*. 5*hr*. 49*m*. 1.632*sec*.
6. What is the value of 0.3355 of a pound avoirdupois? *Ans.* 5*oz*. 5.888*dr*.
7. What is the value of 0.3322 of a ton? *Ans.* 6*cwt*. 2*qr*. 16*lb*. 2.048*oz*.
8. What is the value of 0.2525 of a mile? *Ans.* 2*fur*. 0*rd*. 4*yd*. 1*ft*. 2.4*in*.
9. What is the value of 0.345 of a £? *Ans.* 6*s*. 10*d*. 3.2*far*.
10. What is the value of 0.121212 of a day? *Ans.* 2*hr*. 54*m*. 32.7168*sec*.
11. What is the value of 0.3456 of a £? *Ans.* 6*s*. 10*d*. 3.776*far*.
12. What is the value of 0.9875 of a £? *Ans.* 19*s*. 9*d*.

REDUCTION OF CURRENCIES.

94. Before the adoption of Federal money in this country, accounts were generally kept in the denominations of English money. Different States considered the pound as having different values, as given in the following

TABLE.

\$1 in	England	$=4s.\ 6d.=£\frac{9}{40}$, called Sterling money.
\$1 in	South Carolina Georgia	$=4s.\ 8d.=£\frac{7}{30}$, called Georgia Currency.
\$1 in	Canada Nova Scotia	$=5s.=£\frac{1}{4}$, called Canada Currency.
\$1 in	New England States Virginia Kentucky Tennessee	$=6s.=£\frac{3}{10}$, called New England Currency.
\$1 in	New Jersey Pennsylvania Delaware Maryland	$=7s.\ 6d.=£\frac{3}{8}$, called Pennsylvania Currency.
\$1 in	New York Ohio North Carolina	$=8s.=£\frac{2}{5}$, called New York Currency.

How were accounts kept before the adoption of Federal money? Did all the States estimate the pound at the same value? What fraction of a £ is \$1 in Sterling money? What part of a £ is \$1 in Georgia currency? What part of a £ is \$1 Canada currency? What part of a £ is \$1 New England currency? What part in Pennsylvania currency? What part in New York currency?

CASE I.

95. To reduce Federal money to pounds, shillings, and pence, we obviously have this

RULE.

Multiply the sum in Federal money, by the fractional value of \$1 expressed in pounds, as given in the above Table; the product will be pounds. If there are decimals of a pound, they must be reduced to shillings and pence by Rule under Art. **93**.

What is the fraction by which we multiply Federal money to reduce it to Sterling money? What fraction do we multiply to reduce it to Georgia currency? What is the fraction for Canada currency? What for New England currency? What for Pennsylvania currency? What for New York currency? If in the product there are decimals of a pound, how do you dispose of them?

EXAMPLES.

1. Reduce $100.20 to the different currencies, as given in the preceding Table.

	£	s.	d.	
Ans. $100.20=	22	10	$10\frac{4}{5}$	Sterling money.
	23	7	$7\frac{1}{5}$	Georgia currency.
	25	1	0	Canada currency.
	30	1	$2\frac{2}{5}$	New England currency.
	37	11	6	Pennsylvania currency.
	40	1	$7\frac{1}{5}$	New York currency.

2. Reduce $37.37 to the different currencies.

	£.	s.	d.	
Ans. $37.37=	8	8	1.98	Sterling money.
	8	14	4.72	Georgia currency.
	9	6	10.2	Canada currency.
	11	4	2.64	New England currency.
	14	0	3.3	Pennsylvania currency.
	14	18	11.52	New York currency.

3. Reduce $1000 to equivalent values in the different currencies.

	£		
Ans. $1000=	225		Sterling money.
	233	6s. 8d.	Georgia currency.
	250		Canada currency.
	300		New England currency.
	375		Pennsylvania currency.
	400		New York currency.

CASE II.

96. To reduce a sum in either of the above currencies to Federal money.

It is obvious, that by inverting the fractions which express the value of $1 in pounds, as given in the preceding Table, we shall obtain the value of £1 in dollars. Consequently, we deduce this

RULE.

I. Reduce the shillings and pence, if any, to a decimal of a pound, by Rule under Art. **93.**

II. Multiply the pounds and decimals, if any, by the fractions of the preceding table, after inverting them; the products will be in dollars and decimals of a dollar.

By what fraction must we multiply Sterling money to reduce it to Federal money? What fraction do we multiply by to reduce Georgia currency to Federal money? By what do we multiply to reduce Canada currency? By what to reduce New England currency? By what to reduce Pennsylvania currency? By what to reduce New York currency?

EXAMPLES.

1. Reduce £75 15*s*. 6*d*. of the respective currencies mentioned in the preceding Table, to Federal money.

£75 15*s*. 6*d*.=£75.775, which multiplied by the respective fractions $\frac{40}{9}$, $\frac{30}{7}$, $\frac{4}{1}$, $\frac{10}{3}$, $\frac{8}{3}$, and $\frac{5}{2}$, gives the following answer:

Ans. £75 15*s*. 6*d*.	Sterling money	=\$336.77$\frac{7}{9}$
	Georgia currency	= 324.75
	Canada currency	= 303.10
	New-England currency	= 252.58$\frac{1}{3}$
	Pennsylvania currency	= 202.06$\frac{2}{3}$
	New-York currency	= 189.43$\frac{3}{4}$

2. Reduce £80 5*s*. 3*d*. of the different currencies to Federal money.

Ans. £80 5*s*. 3*d*.	Sterling money	=\$356.722$\frac{2}{9}$
	Georgia currency	= 343.982$\frac{1}{7}$
	Canada currency	= 320.05
	New-England currency	= 267.541$\frac{2}{3}$
	Pennsylvania currency	= 214.033$\frac{1}{3}$
	New-York currency	= 200.656$\frac{1}{4}$

3. Reduce £1000 of the different currencies to Federal money.

Ans. £1000	Sterling money	=\$4444.444$\frac{4}{9}$
	Georgia currency	= 4285.714$\frac{2}{7}$
	Canada currency	= 4000.
	New-England currency	= 3333.333$\frac{1}{3}$
	Pennsylvania currency	= 2666.666$\frac{2}{3}$
	New-York currency	= 2500.

The following are the rates at which some of the foreign

coins are estimated at the custom-houses of the United States:

English £, by act of Congress of 1832 -	$4.80
Livre of France - - - - - -	0.18\frac{1}{2}$
Franc do. - - - - - - - -	0.18\frac{3}{4}$
Silver Rouble of Russia - - - - -	$0.75
Florin or Guilder of the United Netherlands -	$0.40
Mark Banco of Hamburg - - - -	0.33\frac{1}{3}$
Real of Plate of Spain - - - -	$0.10
Real of Vellon of do. - - - - -	$0.05
Milrea of Portugal - - - - - -	$1.24
Tale of China - - - - - -	$1.48
Pagoda of India - - - - - -	$1.84
Rupee of Bengal - - - - - -	$0.50

RULE OF THREE.

97. The quotient arising from dividing one quantity by another of the same *kind* or *denomination*, is called a *ratio*.

Thus, the ratio of

$$12 \text{ to } 2=\tfrac{12}{2}=6$$
$$12 \text{ to } 3=\tfrac{12}{3}=4$$
$$12 \text{ to } 4=\tfrac{12}{4}=3$$
$$12 \text{ to } 6=\tfrac{12}{6}=2$$
$$12 \text{ to } 12=\tfrac{12}{12}=1$$

Hence, we see that the ratio of two quantities shows how many times greater the one is than the other. It is therefore evident, that there can not exist a ratio between two quantities of different denominations. There is no ratio between 12 feet and 3 pounds, for we can not say how many times 12 feet is greater than 3 pounds. But there is a ratio between 12 feet and 3 feet, which is

$$\frac{12ft.}{3ft.}=4.$$

There is the same ratio between 12 pounds and 3

pounds. The ratio is itself an abstract number; it is not a denominate number. The ratio of 12 feet to 3 feet is 4 units simply; it is neither 4 feet nor 4 pounds, but simply 4 times 1; showing that 12 feet is 4 times as great as 3 feet. In this way we find

The ratio of 10 yards to 5 yards $=\frac{10}{5}=2$.
" 8 inches to 4 inches $=\frac{8}{4}=2$.
" 7 ounces to 3 ounces $=\frac{7}{3}=2\frac{1}{3}$.
" 5 bushels to 2 bushels $=\frac{5}{2}=2\frac{1}{2}$.
" 7 rods to 4 rods $=\frac{7}{4}=1\frac{3}{4}$.
" 9 cords to 4 cords $=\frac{9}{4}=2\frac{1}{4}$.
" 40 acres to 18 acres $=\frac{40}{18}=\frac{20}{9}=2\frac{2}{9}$.

When the ratio of two quantities is the same as the ratio of two other quantities, the four quantities are in *proportion*. Thus, the ratio of 8 yards to 4 yards, is the same as the ratio of 12 dollars to 6 dollars; therefore, there is a proportion between 8 yards, 4 yards, 12 dollars, and 6 dollars.

The usual method of denoting that four terms are in proportion, is by means of points, or dots. Thus, the above proportion is written

8 yards : 4 yards : : 12 dollars : 6 dollars.

Where two dots are placed between the first and second terms, and between the third and fourth; and four dots are placed between the second and third.

The above proportion is read

8 yards is to 4 yards as 12 dollars is to 6 dollars.

Of the four terms constituting a proportion, the first and fourth are called *extremes;* the second and third are called *means*.

Since the quotient of the first term of a proportion divided by the second, is equal to the quotient of the third term divided by the fourth; it follows that *the product of the extremes is equal to the product of the means.*

Hence, if the product of the extremes be divided by either mean, the quotient will be the other mean.

Also, if the product of the means be divided by either extreme, the quotient will be the other extreme.

From the above properties, we see that if any *three*

of the four terms which constitute a proportion are given, the remaining term can be found.

98. The method of finding the fourth term of a proportion, when *three* terms are given, constitutes the RULE OF THREE.

What is the quotient arising from dividing one number by another of the same kind called? What is the ratio of 12 to 2? Of 12 to 3? Of 12 to 4? What does the ratio of two quantities show? Can a ratio exist between two quantities of different denominations? Is there a ratio between 12 feet and 3 pound? What is the ratio of 12 feet to 3 feet? Can the ratio be a denominate number? What is the ratio of 10 yards to 5 yards? Of 8 inches to 4 inches? How are four quantities related when the ratio of the first to the second is the same as the ratio of the third to the fourth? When four terms are in proportion, how are they written? Of the four terms constituting a proportion, which are called extremes? Which are called means? To what is the product of the extremes equal? If the product of the extremes be divided by one of the means, what will the quotient be? If the product of the means be divided by one of the extremes, what will the quotient be? How many terms of a proportion must be known in order to find the others?

1. Let us endeavor to find the value of 24 yards of cloth, on the supposition that 8 yards are worth \$12.

It is obvious that the value sought must be as many times greater than \$12 as 24 yards is greater than 8 yards. Hence, there is the same ratio between \$12 and the *value sought*, as there is between 8 yards and 24 yards. Consequently, we have this proportion:

8 yards : 24 yards : : \$12 : *value sought.*

Taking the product of the means, we have $24 \times 12 = 288$. This, divided by the first term, gives $\frac{288}{8} = 36$ for the fourth term sought, which must be of the same kind as the third term; therefore, \$36 is the value of 24 yards.

2. What will 312 pounds of coffee cost, if 25 pounds cost \$3.25?

In this example, the ratio of 25 pounds to 312 pounds, is the same as the ratio of \$3.25 to the number of dollars sought. Hence,

25 pounds : 312 pounds : : $3.25 : the answer.

```
                 312
                 ---
                 650
                325
               975
               -----
        25)1014.00($40.56
           100
           ---
            140
            125
            ---
             150
             150
             ---
```

Here we first multiply the means together; we then divide the product by the first term.

Since there is a ratio between the third and fourth terms, it follows that they must be of the same denominate value. Hence, of the three terms given, we may always take for the third term of our proportion the one which is of the same kind as the answer required; then, if the answer sought is to be greater than this third term, the second term must exceed the first; but if the answer sought is to be less than this third term, then the second term must be less than the first.

99. From what has been said and done, we deduce this first form for the

RULE OF THREE.

Form a proportion by placing for the third term that which is of the same kind as the answer sought; the two remaining terms must be taken for the first and second terms, observing to take the larger for the second term, when the answer sought is to exceed the third term; but take the smaller of the two for the second term, when the answer is to be less than the third term.

Having written the three terms of the proportion, or, as usually expressed, having stated the question, then multiply the second and third terms together, and divide the product by the first term.

NOTE.—Since there is a ratio between the first and cond terms, they must be reduced to the same denominate lue. Also, the third term must be reduced to its lowest nomination; then the quotient found by dividing the oduct of the means by the first term, will be of the same nomination as the third term.

In stating questions in the Rule of Three, which term must be ken for the third? Of the two remaining terms, which is to be ken for the second? After the question is stated, how do you oceed to find the answer? Is it ever necessary to make any eduction in the terms before multiplying and dividing? What are hese reductions? The answer, when found, will be of the same ame as which term?

EXAMPLES.

1. What is the cost of 6 cords of wood, at $7 for 2 cords?

2 cords : 6 cords : : $7 : *Ans.*

$$\begin{array}{r} 7 \\ \underline{6} \\ 2)\underline{42} \\ \textit{Ans.}\ \$21 \end{array}$$

2. What will 9 pair of shoes cost, if 5 pair cost £2 2*s*. 6*d*.?

5 pair : 9 pair : : £2 2*s*. 6*d*.

When reduced, 5 pair : 9 pair : : 510*d*.

$$\begin{array}{r} 510 \\ \underline{9} \\ 5)\underline{4590} \\ \textit{Ans.}\ 918d. = £3\ 16s.\ 6d. \end{array}$$

3. If there are 9 weeks in 63 days, how many weeks are there in 365 days?

63 days : 365 days : : 9 weeks.

$$\begin{array}{l} \quad\ \ 365 \\ \quad\ \ \underline{\ \ \ 9} \\ 63)3285(52\tfrac{9}{63} = 52\tfrac{1}{7} \text{ weeks. } \textit{Ans.} \\ \quad\ \ \underline{315} \\ \quad\ \ \ \ 135 \\ \quad\ \ \ \ \underline{126} \\ \quad\ \ \ \ \ \ \ 9 \end{array}$$

4. If 12 barrels of flour are worth \$54, what is the value of 42 barrels at the same rate?

$$
\begin{array}{r}
12 \text{ barrels} : 42 \text{ barrels} :: \$54 \\
\underline{42} \\
108 \\
\underline{216} \\
12)2268(189 \text{ dollars. } \textit{Ans.} \\
\underline{12} \\
106 \\
\underline{96} \\
108 \\
\underline{108}
\end{array}
$$

In this example, it is obvious that 2 times 12 barrels would be worth 2 times \$54; 3 times 12 barrels would be worth 3 times \$54; 4 times 12 barrels would be worth 4 times \$54, and so on for other ratios. The ratio of 42 barrels to 12 barrels is $\frac{42}{12}$.

Now, if we multiply \$54 by this ratio, it will evidently give the value of 42 barrels. The operation may be expressed as follows:

$$\$54 \times \tfrac{42}{12}.$$

We may now employ the same rules for simplifying this expression as were used under ART. 73. That is to say, we may reject such factors as are common to both numerators and denominators. Thus, dividing the denominator 12, and the numerator 42, each by 6, it becomes

$$\$54 \times \frac{\overset{7}{\cancel{42}}}{\underset{2}{\cancel{12}}} \text{ or, } \$54 \times \tfrac{7}{2}.$$

Now, dividing the denominator 2 and \$54 of numerator each by 2, we have

$$\overset{27}{\cancel{\$54}} \times \frac{7}{\cancel{2}} \text{ or, } \$27 \times 7 = \$189 \textit{ Ans.}$$

5. What will 84 bushels of apples cost, if 14 bushels are worth \$6.75?

The ratio of 84 bushels to 14 bushels is $\frac{84}{14}$. Now, multiplying \$6.75 by this ratio, we have

$$\$6.75 \times \tfrac{84}{14}.$$

Dividing 84 of numerator and 14 of the denominator each by 7, we obtain

$$\$6.75 \times \frac{\overset{12}{\cancel{84}}}{\underset{2}{\cancel{14}}} \text{ or, } \$6.75 \times \tfrac{12}{2}.$$

Again, dividing 12 of numerator and 2 of denominator each by 2,

$$\$6.75 \times \frac{\overset{6}{\cancel{12}}}{\cancel{2}} \text{ or, } \$6.75 \times 6 = \$40.50. \textit{ Ans.}$$

From these two examples, we see that the rule of three may be given in the following simple form:

RULE OF THREE.

Of the three terms which are given, one will always be of the same kind as the answer sought; this will be the third term. Then, if by the nature of the question, the answer is required to be greater than the third term, divide the greater of the two remaining terms by the less, for a ratio; but if the answer is required to be less than the third term, then divide the less of the two remaining terms by the greater, for a ratio. Having obtained the ratio, multiply the third term by it, and it will give the answer in the same denomination as was the third term.

Note.—Before obtaining the ratio, by means of the first two terms, we must reduce them to like denominations.

6. If 200 sheep yield 650 pounds of wool, how many pounds will 825 sheep yield?

In this example, the answer is required to be in pounds;

we therefore take 650 pounds for the third term. The ratio of 825 sheep to 200 sheep is $\frac{825}{200}$. Hence, we have

$$650lb. \times \frac{825}{200}.$$

Cancelling, we have

$$650lb. \times \frac{\overset{33}{\cancel{825}}}{\underset{8}{\cancel{200}}} \text{ or, } 650lb. \times \frac{33}{8}.$$

Again, cancelling, we have

$$325lb. \times \frac{33}{\underset{4}{\cancel{8}}} = \frac{325 \times 33}{4} = 2681\tfrac{1}{4}lb.$$

7. If $\frac{11}{13}$ of a pound of sugar cost $\frac{23}{26}$ of a shilling, how much will $\frac{9}{23}$ of a pound cost?

In this example, our third term is $\frac{23}{26}$ of a shilling. And since $\frac{9}{23}$ of a pound is less than $\frac{11}{13}$, we must obtain our ratio by dividing $\frac{9}{23}$ by $\frac{11}{13}$, which gives $\frac{9}{23} \times \frac{13}{11}$; this, multiplied by the third term, $\frac{23}{26}$ of a shilling, will give $\frac{23}{26}$ of a shilling $\times \frac{9}{23} \times \frac{13}{11}$. To reduce this with the least labor, we must resort to the method of cancelling. Thus, cancelling the 23, which occurs in both numerator and denominator, also 13 of the numerator against a part of the 26 of the denominator, our expression will, by this means, become $\frac{1}{2}$ of a shilling $\times \frac{9}{1} \times \frac{1}{11} = \frac{9}{22}$ of a shilling.

NOTE.—This method of cancelling should be used when the nature of the question will admit, since it will always simplify the operation.

8. If a tree 38 feet 9 inches in height, give a shadow of 49 feet 2 inches, how high is that tree, which, at the same time, casts a shadow of 71 feet 7 inches?

In this example, our third term is the height of the first tree, which is 38 feet 9 inches $= 38\frac{3}{4}$ feet $= \frac{155}{4}$ feet: our ratio will be obtained by dividing 71 feet 7 inches $= 71\frac{7}{12}$ feet $= \frac{859}{12}$ feet, by 49 feet 2 inches $= 49\frac{1}{6}$ feet $= \frac{295}{6}$ feet: which becomes $\frac{859}{12} \times \frac{6}{295}$; this multiplied by the third term, $\frac{155}{4}$ feet, gives $\frac{155}{4}$ feet $\times \frac{859}{12} \times \frac{6}{295}$. Cancelling 6 of the numerator against a part of the 12 of the denomina-

tor, also cancelling 5, a factor of 155 of the numerator, against 5, a factor of 295 of the denominator, we get $\frac{31}{4}$ feet $\times \frac{859}{2} \times \frac{1}{59} = \frac{26629}{472} = 56\frac{197}{472}$ feet, for the answer.

9. If $3\frac{1}{2}$ pounds of coffee cost $2\frac{1}{3}$ shillings, how much will $10\frac{1}{6}$ pounds cost?

In this example, $2\frac{1}{3} = \frac{7}{3}$ shillings must be our third term; and since $10\frac{1}{6} = \frac{61}{6}$ pounds must cost more than $3\frac{1}{2} = \frac{7}{2}$ pounds, we must divide $\frac{61}{6}$ by $\frac{7}{2}$ for the ratio; making it $\frac{61}{6} \times \frac{2}{7}$; this, multiplied by the third term, $\frac{7}{3}$ shillings, will give $\frac{7}{3}$ shillings $\times \frac{61}{6} \times \frac{2}{7}$; which becomes, after cancelling, $\frac{1}{3}$ of a shilling $\times \frac{61}{3} = \frac{61}{9}$ shillings $= 6\frac{7}{9}$ shillings.

10. Gave $72 for 11 barrels of fish. How much will 88 barrels cost at the same rate? *Ans.* $576.

11. If $43\frac{1}{3}$ pounds of cheese cost $2.20, what will $216\frac{2}{3}$ pounds cost at the same rate? *Ans.* $11.

12. If I pay $3.90 for sawing 7 cords of wood, how much ought I to give for sawing $23\frac{1}{3}$ cords? *Ans.* $13.

13. If $\frac{3}{10}$ of a ship is worth $2853, what is the whole worth?

The ratio of the whole ship, or $\frac{10}{10}$, to $\frac{3}{10}$, is $\frac{10}{3}$. Hence, $\$2853 \times \frac{10}{3} = \$951 \times 10 = \$9510$ *Ans.*

14. If $\frac{4}{13}$ of my income is $533, what is my whole income? *Ans.* $1732.25.

15. A person failing in business, finds that he owes $7560, and that he only has $3100 to pay it with. How much can he pay to that creditor whose claim is $756? *Ans.* $310.

16. If it require $5\frac{1}{2}$ bushels of wheat to make one barrel of flour, how many bushels will it require for 100 barrels of flour? *Ans.* 550 bushels.

17. If 7 barrels of flour are sufficient for a family 6 months, how many barrels will they require for 11 months? *Ans.* $12\frac{5}{6}$ barrels.

18. If it take 25 yards of carpeting a yard wide to cover a certain floor, how many yards of $\frac{3}{4}$ carpeting would be necessary to cover the same floor? *Ans* $33\frac{1}{3}$ yards.

19. If a person travel 8 miles in 10 hours, how far will he travel in 5 days, by travelling 8 hours each day? *Ans.* 32 miles

20. If 35 pounds of feathers cost \$15, what will 100 pounds cost at the same rate? *Ans.* \$42.85$\frac{5}{7}$.

21. If a man perform a certain piece of work in 18 days, when he works 8 hours per day, how many days will he require if he work 10 hours each day?

Ans. 14 days 4 hours.

22. If a piece of board 12 inches wide and 12 inches long make one square foot, how many inches of length must be taken from a board 15 inches wide to make a square foot? *Ans.* 9$\frac{3}{5}$ inches.

23. If 8 men can mow a field in 5 days, in how many days can 5 men do the same? *Ans.* 8 days.

24. If 27$\frac{1}{2}$ yards of cloth cost \$60, how many yards can I buy for \$100? *Ans.* 45$\frac{5}{6}$ yards.

25. If 27$\frac{1}{2}$ yards of cloth cost \$60, what will 45$\frac{5}{6}$ yards cost? *Ans.* \$100.

26. If $\frac{5}{8}$ of a ship is worth \$9000, what is her whole value?

The whole ship being a unit, or $\frac{8}{8}$, we have the ratio $\frac{8}{5}$; hence, the answer is $\$9000\times\frac{8}{5}=\$14400$.

27. If $\frac{3}{16}$ of a city lot is sold for \$500, what would $\frac{7}{16}$ of the same lot sell for at the same rate? *Ans.* \$1166$\frac{2}{3}$.

28. Admitting that the earth moves in its orbit about the sun, a distance of 597000000 miles, in 365 days 6 hours, how far on an average does it move in each hour?

Ans. 68104$\frac{56}{1461}$ miles.

29. The equatorial portions, by the diurnal rotation of the earth, move about 24900 miles each day? How far is that in each hour? *Ans.* 1037$\frac{1}{2}$ miles.

30. If it require 10 years of 365$\frac{1}{4}$ days for light to pass from a fixed star to the earth, how many miles distant is it, on the supposition that light moves 192000 miles in 1 second? *Ans.* 60590592000000 miles.

31. If by a leak of a ship $\frac{3}{5}$ enough water run in in 4 hours to sink her, in what time must she sink?

Ans. 6*hr.* 40*m.*

32. If I pay \$25 for the masonry of 4000 brick, how much ought I to pay for the work which requires 100000 brick? *Ans.* \$625.

33. If a steam-ship require 14 days to sail a distance of 3000 miles, what time, at the same rate of sailing, would she require to sail 24900 miles?

Ans. 116 days $4\frac{4}{5}$ hours.

34. Admitting the diameter of the earth to be 8000 miles, and the highest mountain to be 5 miles, what elevation must be made on a globe of 16 inches diameter to represent accurately the height of such mountain?

Ans. $\frac{1}{100}$ of an inch.

35. If $100 in 12 months bring an interest of $7, how much will be the interest of $100 for 8 months?

Ans. 4.66\frac{2}{3}$.

36. If the interest of $100 for 12 months is $7, what will be the interest of $75 for the same time?

Ans. $5.25.

37. If in 12 months the interest of $100 is $7, how long must $100 be on interest to gain $10?

Ans. $17\frac{1}{7}$ months.

38. If a glacier of 60 miles in length move 50 inches per annum, in what time will it move its whole length?

Ans. 76032 years.

39. If a staff of 10 feet in length give a shadow of 15 feet, how high is that tree whose shadow measures 90 feet? *Ans.* 60 feet.

40. Suppose sound to move 1100 feet in a second; how many miles distant is a cloud, in which lightning is observed 16 seconds before the thunder is heard, no allowance being made for the motion of light? *Ans.* $3\frac{1}{3}$ miles.

41. If it require 30 yards of carpeting which is $\frac{3}{4}$ of a yard wide to cover a floor, how many yards of carpeting which is $1\frac{1}{4}$ yards wide, will be necessary to cover the same floor? *Ans.* 18 yards.

COMPOUND PROPORTION.

100. When the quantity required depends upon more than three terms, the operation of finding it is called the *rule of compound proportion*, which may be thus given:

RULE.

Among the terms given there will be but one like the answer, which we will call the odd term. The other terms will appear in pairs or couplets. Form ratios out of each couplet in the same manner as in the rule of three; then multiply all the ratios and the odd term together, and it will give the answer in the same name and denomination as the odd term.

NOTE.—Before forming ratios from the couplets, they must be reduced to the same denominate value.

EXAMPLES.

1. If a person travel 300 miles in 17 days, travelling only 6 hours each day, how many miles could he have gone in 15 days, by travelling 10 hours each day?

In this example, the answer is required in miles, therefore our odd term is 300 miles.

The first couplet consists of days; and since in 15 days, other things being the same, he could not travel as far as in 17 days, we must divide 15 by 17, which gives $\frac{15}{17}$ for the first ratio.

The second couplet consists of hours; and since in 10 hours he could travel farther than in 6 hours, we must divide 10 by 6, which gives $\frac{10}{6}$ for the second ratio.

Multiplying these two ratios and the odd term together, we get 300 miles$\times\frac{15}{17}\times\frac{10}{6}$. Cancelling the 6 of the denominator against a part of 300 of the numerator, it becomes 50 miles$\times\frac{15}{17}\times\frac{10}{1}=441\frac{3}{17}$ miles, for the answer.

2. If a marble slab 10 feet long, 3 feet wide, and 3 inches thick, weigh 400 pounds, what will be the weight of another slab, of the same marble, whose length is 8 feet, width 4 feet, and thickness 5 inches?

In this example, the answer is required to be given in pounds; therefore 400 pounds is the odd term. The first couplet consists of the lengths; and since 8 feet in length will give less weight than 10 feet, we must divide 8 by 10, which gives $\frac{8}{10}$ for the first ratio.

The second couplet consists of the widths; and since 4

feet wide will give more weight than 3 feet, we must divide 4 by 3, which gives $\frac{4}{3}$ for the second ratio.

The third couplet consists of thicknesses; and since 5 inches thick will give more weight than 3 inches, we must divide 5 by 3, which gives $\frac{5}{3}$ for the third ratio.

Multiplying the odd term and these ratios together, we get 400lbs. $\times \frac{8}{10} \times \frac{4}{3} \times \frac{5}{3}$. Cancelling the 10 of the denominator against a part of the 400 of the numerator, we get 40lbs. $\times \frac{8}{1} \times \frac{4}{3} \times \frac{5}{3} = \frac{6400}{9} = 711\frac{1}{9}$ pounds, for the answer.

3. 500 men, working 12 hours each day, have employed 57 days to dig a canal of 1800 yards long, 7 yards wide, and 3 yards deep; how many days must 860 men, working 10 hours each day, employ in digging another canal of 2900 yards long, 12 yards wide, and 5 yards deep, in a soil which is 3 times as difficult to excavate as the first?

In this example, the odd term is 57 days.

The different ratios will be as follows:

$\frac{500}{860} = \frac{25}{43}$ ratio of the men.
$\frac{12}{10} = \frac{6}{5}$ ratio of the hours.
$\frac{2900}{1800} = \frac{29}{18}$ ratio of lengths of the canals.
$\frac{12}{7} =$ ratio of widths of the canals.
$\frac{5}{3} =$ ratio of depths of the canals.
$\frac{3}{1} =$ ratio of the difficulty in excavation.

Multiplying successively these ratios and the odd term, we have

$$57 \text{ days} \times \frac{25}{43} \times \frac{6}{5} \times \frac{29}{18} \times \frac{12}{7} \times \frac{5}{3} \times \frac{3}{1}.$$

This becomes, after cancelling factors,

$$19 \text{ days} \times \frac{5}{43} \times \frac{6}{1} \times \frac{29}{1} \times \frac{2}{7} \times \frac{5}{1} \times \frac{1}{1} = 549\frac{51}{301} \text{ days.}$$

4. 15 men, working 10 hours each day, have employed 18 days to build 450 yards of stone fence; how many men, working 12 hours each day, for 8 days, will be requisite to build 480 yards of similar fence? *Ans.* 30 men.

5. If it require 1200 yards of cloth $\frac{5}{4}$ wide to clothe 500 men, how many yards which is $\frac{7}{8}$ wide will it take to clothe 960 men? *Ans.* $3291\frac{3}{7}$ yards.

6. If 8 men will mow 36 acres of grass in 9 days, by working 9 hours each day, how many men will be re-

quired to mow 48 acres in 12 days, by working 12 hours each day? *Ans.* 6 men.

7. If 11 men can cut 49 cords of wood in 7 days, when they work 14 hours per day, how many men will it take to cut 140 cords in 28 days, by working 10 hours each day? *Ans.* 11 men.

8. If 12 ounces of wool make $2\frac{1}{2}$ yards of cloth, that is 6 quarters wide, how many pounds of wool will it take for 150 yards of cloth, 4 quarters wide? *Ans.* 30 pounds.

9. If the wages of 6 men for 14 days be 84 dollars, what will be the wages of 9 men for 16 days? *Ans.* $144.

10. If 100 men in 40 days of 10 hours each, build a wall 30 feet long, 8 feet high, and 2 feet thick, how many men must be employed to build a wall 40 feet in length, 6 feet high, and 4 feet thick, in 20 days, by working 8 hours each day? *Ans.* 500 men.

11. In how many days, working 9 hours a day, will 24 men dig a trench 420 yards long, 5 yards wide, and 3 yards deep, if 248 men, working 11 hours a day, in 5 days, dig a trench 230 yards long, 3 yards wide, and 2 yards deep? *Ans.* $288\frac{59}{207}$ days.

12. Suppose that 50 men, by working 5 hours each day, can dig, in 54 days, 24 cellars, which are each 36 feet long, 21 feet wide, and 10 feet deep, how many men would be required to dig, in 27 days, 18 cellars, which are each 48 feet long, 28 feet wide, and 9 feet deep, provided they work only 3 hours each day? *Ans.* 200 men.

PRACTICE.

101. Practice is a short method of finding the answer to such questions in the *Rule of Three* as have a unit for their first term.

As an example, suppose one bushel of apples to be worth 50 cents, what is the value of $18\frac{1}{4}$ bushels?

Had the apples been worth $1 per bushel, it is plain

that 18½ bushels would have been worth \$18½, that is, \$18.50. Now since 50 cents is just half of one dollar, they must have been worth half of \$18.50=\$9.25.

In order to work by this rule, we must make use of *aliquot parts*. An aliquot part of anything is an exact part; in the above example 50 cents is an aliquot part of \$1, since it is exactly half of \$1. We will give some aliquot parts which are in frequent use, in the following

TABLE OF ALIQUOT PARTS.

cts.	\$	*mo.*	*yr.*	*s.*	£	*d.*	*s.*
50 =	$\frac{1}{2}$	6 =	$\frac{1}{2}$	10 =	$\frac{1}{2}$	6 =	$\frac{1}{2}$
33⅓ =	$\frac{1}{3}$	4 =	$\frac{1}{3}$	6 8*d.* =	$\frac{1}{3}$	4 =	$\frac{1}{3}$
25 =	$\frac{1}{4}$	3 =	$\frac{1}{4}$	5 =	$\frac{1}{4}$	3 =	$\frac{1}{4}$
20 =	$\frac{1}{5}$	2 =	$\frac{1}{6}$	4 =	$\frac{1}{5}$	2 =	$\frac{1}{6}$
12½ =	$\frac{1}{8}$	1 =	$\frac{1}{12}$	3 4*d.* =	$\frac{1}{6}$	1½ =	$\frac{1}{8}$
10 =	$\frac{1}{10}$	15*da.* =	$\frac{1}{2}$ of 1 *mo.*	2 6*d.* =	$\frac{1}{8}$	1 =	$\frac{1}{12}$ *d.*
6¼ =	$\frac{1}{16}$	10 =	$\frac{1}{3}$	2 =	$\frac{1}{10}$	2 *far.* =	$\frac{1}{2}$ of 1
5 =	$\frac{1}{20}$	5 =	$\frac{1}{6}$	1 8*d.* =	$\frac{1}{12}$	1 =	$\frac{1}{4}$

What is Practice? What is an aliquot part of any thing? Repeat all the aliquot parts of a dollar as given in the above Table. Repeat in the same way all the other aliquot parts of the Table.

EXAMPLES.

1. What will 435 yards of cloth cost, at \$0.75 per yard?

435 yards, at \$1 per yard=\$435
435 yards, at 50 cents per yard=½ of \$435=217.50
435 yards, at 25 cents per yard=¼ of \$435=108.75
435 yards, at 75 cents per yard= \$326.25

2. What cost 13½ pounds of tea, at 5*s*. 6*d*. per pound?

13*lb*. at 5*s*.	=65*s*.=£3	5*s*.	
13*lb*. at 6*d*. or, ½*s*.=		6	6*d*.
½*lb*. at 5*s*. =		2	6
½*lb*. at 6*d*. =			3
	£3	14*s*.	3*d*.

3. What cost 37½ dozen of eggs, at 1*s*. 4*d*. per dozen?

37*doz*. at 1*s*. per *doz*.	=37*s*.=	£1	17*s*.	
37*doz*. at 4*d*. or ⅓*s*.	=12⅓=		12	4*d*.
½*doz*. at 1*s*.	=			6
½*doz*. at 4*d*.	=			2
	Ans.	£2	10*s*.	

4. What cost 7½ cords of wood, at $2.75 per cord? $20.625

5. What is the value of 28¾ pounds of butter, at 11 cents per pound? *Ans*. $3.1625.

6. What is the value of 500½ yards of tape, at 2¼ cents per yard? *Ans*. $11.26125.

7. What must I give for 13¾ bushels of oats, at 2*s*. 4*d*. per bushel? *Ans*. £1 12*s*. 1*d*.

8. What cost 18¾ pounds of ham at 8 cents per pound? *Ans*. $1.50.

9. What cost 15¾ gallons of oil, at $0.75 cents per gallon? *Ans*. $11.8125.

10. What cost 4000 quills, at $2.25 per 1000? *Ans*. $9.

11. What cost 27¾ yards of carpeting, at 6*s*. 6*d*. per yard? *Ans*. £9 0*s*. 4½*d*.

12. What is the value of 25 bushels of potatoes, at $0.31¼ per bushel? *Ans*. $7.8125.

13. What is the value of 54 spelling-books, at 12½ cents per copy? *Ans*. $6.75.

14. What is the value of 47½ reams of paper, at $3.25 per ream? *Ans*. $154.375.

15. What is the value of 30½ gross of almanacs, at $2.25 per gross? *Ans*. $68.625.

16. What cost 16¾ gallons of vinegar, at 1*s*. 4*d*. per gallon? *Ans*. £1 2*s*. 4*d*.

17. What is the value of 5⅓ bushels of walnuts, at 8*s*. 6*d*. per bushel? *Ans*. £2 5*s*. 4*d*.

18. What cost 3½ gross of matches, at $1.125 per gross? *Ans*. $3.9375.

19. What cost 325 bushels of apples, at $37\frac{1}{2}$ cents per bushel? *Ans.* \$121.875.

20. What cost $16\frac{1}{2}$ yards of cloth, at $\$3\frac{3}{4}$ per yard? *Ans.* \$61.875.

SIMPLE INTEREST.

102. INTEREST is money paid by the borrower to the lender, for the use of the money borrowed.

It is estimated at a certain rate *per cent per annum,* that is, a certain number of dollars for the use of \$100, for one year.

Thus, when \$6 is paid for the use of \$100, for one year, the interest is said to be at 6 *per cent.*

In the same manner when \$5 is paid for the use of \$100, for one year, the interest is said to be at 5 *per cent.;* and the same for other rates.

The rate *per cent.* is generally fixed by law. In the New England States the legal rate is 6 *per cent.*, while in the State of New York it is 7 *per cent.*

The sum of money borrowed, or upon which the interest is computed, is called the *principal.*

The principal, with the interest added to it, is called the *amount.*

What is Interest? How is it estimated? What is the rate per cent. when \$6 is paid for the use of \$100, for one year? What is the rate per cent. when \$5 is in the same way paid? Is the rate per cent. generally fixed by law? What is the legal rate per cent. in the New England States? What is it in the State of New York? What is the principal? What is the amount?

CASE I.

To find the interest on \$1, for any given time, at 6 *per cent.*

The interest on \$100, for one year, at 6 per cent. being \$6, it follows that the interest on \$1, for one year, is \$0.06; and since 2 months is $\frac{2}{12}=\frac{1}{6}$ of a year, the interest on \$1, for 2 months, is \$0.01; again, since 6 days is $\frac{6}{60}=\frac{1}{10}$ o:

2 months, when we reckon 30 days to each month, it follows that the interest on \$1, for 6 days, is \$0.001. Hence, we have the following

RULE.

Call half the number of months, CENTS; *one sixth the number of days,* MILLS.

EXAMPLES.

1. What is the interest of \$1, for 7 months and 10 days, at 6 per cent.?

7 months gives	\$0.035
10 days gives	$1\frac{2}{3}$
Ans.	\$0.036$\frac{2}{3}$

2. What is the interest of \$1, for 11 months and 11 days, at 6 per cent.?

11 months gives	\$0.055
11 days gives	$1\frac{5}{6}$
Ans.	\$0.056$\frac{5}{6}$

3. What is the interest of \$1, for 3 years 7 months, that is, for 43 months, at 6 per cent.? *Ans.* \$0.215.
4. What is the interest of \$1, for 2 years 7 months and 9 days, at 6 per cent.? *Ans.* \$0.1565.
5. What is the interest of \$1, for 1 year 7 months and 15 days, at 6 per cent.? *Ans.* \$0.0975.
6. What is the interest of \$1, for 7 years and 9 days, at 6 per cent.? *Ans.* \$0.4215.
7. What is the interest of \$1, for 3 years 5 months and 3 days, at 6 per cent.? *Ans.* \$0.2055.
8. What is the interest of \$1, for 9 years and 3 months, at 6 per cent.? *Ans.* \$0.555.
9. What is the interest of \$1, for 21 years 5 months and 6 days, at 6 per cent.? *Ans.* \$1.286.
10. What is the interest of \$1, for 7 months and 11 days, at 6 per cent.? *Ans.* \$0.036$\frac{5}{6}$.
11. What is the interest of \$1, for 3 years and 9 months, at 6 per cent.? *Ans.* \$0.225.

CASE II.

To find the interest of any given principal, for any given time, at 6 per cent., we have this

RULE.

Find the interest on $1, *for the given time, by Case I.; multiply the interest thus found by the given principal.*

EXAMPLES.

1. What is the interest of $49.37, for 13 months and 15 days, at 6 per cent.?

In this example, we find the interest on $1, for 13 months and 15 days, at 6 per cent., to be $0.0675, which, multiplied by the principal, gives $3.332475, for the interest on $49.37, for the given time.

2. What is the interest of $608.62, for 1 year and 9 months, at 6 per cent.? *Ans.* $63.9051
3. What is the interest of $341.13, for 7 years and 9 days, at 6 per cent.? *Ans.* $143.786295.
4. What is the interest of $100, for 16 years and 8 months, at 6 per cent.? *Ans.* $100.
5. What is the interest of $591.03, for 4 years 3 months and 7 days, at 6 per cent.? *Ans.* $151.402185.
6. What is the interest of $0.134, for 4 months and 3 days, at 6 per cent.? *Ans.* $0.002747.
7. What is the interest of $7.50, for 7 months, at 6 per cent.? *Ans.* $0.2625.
8. What is the interest of $371.01, for 4 years and 15 days, at 6 per cent.? *Ans.* $89.969925.
9. What is the interest of $57.92, for 3 years 7 months and 9 days, at 6 per cent.? *Ans.* $12.53968.
10. What is the interest of $329, for 5 years and 13 days, at 6 per cent.? *Ans.* 99.412\frac{5}{6}$.
11. What is the interest of $47.39, for 1 year and 7 months, at 6 per cent.? *Ans.* $4.50205.

CASE III.

To find the interest on any given principal, for any given time, at any given rate per cent., we have this

RULE.

Find the interest on the given principal, for the given time, at 6 per cent., by Case II. Then increase, or decrease, this interest by the same part of itself, as it would be necessary to increase, or decrease 6, in order to make it agree with the given rate per cent.

EXAMPLES.

1. What is the interest of $19.41, for 1 year 7 months and 13 days, at 7 per cent.?

In this example, we find by Case II., that the interest of $19.41, for 1 year 7 months and 13 days, at 6 per cent., is $1.886005. Since 6, increased by its sixth part, equals 7, it will be necessary to increase the interest just found for 6 per cent., by its sixth part, which becomes 2.200339\frac{1}{6}$, for the interest at 7 per cent.

2. What is the interest of $530, for 3 years and 6 months, at 5 per cent.? *Ans.* $92.75.

In this example, it was necessary to decrease the interest of 6 per cent., by its sixth part.

3. What is the interest of $5.37, for 4 years and 12 days, at 8 per cent.? *Ans.* $1.73272.

In this example we, increase the interest at 6 per cent., by its third part.

4. What is the interest of $4070, for 3 months, at 9 per cent.? *Ans.* $91.575.

5. What is the interest of $3671, for 6 months, at 10 per cent.? *Ans.* $183.55.

6. What is the interest of $4920.05, for 3 months, at 4 per cent.? *Ans.* $49.2005.

7. What is the interest of $40.17, for 3 months and 18 days, at 3 per cent.? *Ans.* $0.36153.

8. What is the interest of $37.13, for 5 months and 12 days, at $4\frac{1}{2}$ per cent.? *Ans.* $0.7518825.

9. What is the interest of $489, for 3 years and 4 months, at $5\frac{1}{2}$ per cent.? *Ans.* $89.65.

10. What is the interest of $700, for 1 year and 9 months, at 7 per cent.? *Ans.* $85.75.

NOTE.—When the principal is given in English money, we must reduce the shillings, pence, and farthings, to the decimal of a £; and then proceed as in Federal Money.

11. What is the interest of £75 13*s.* 6*d.*, for 3 years and 5 months, at 6 per cent.?

In this example, 13*s.* 6*d.*, reduced to the decimal of a £, is 0.675, so that our principal is £75.675; the interest on £1, for 3 years and 5 months, at 6 per cent., is £0.205, which, multiplied into £75.675, gives £15.513375=£15 10*s.* $3\frac{21}{100}$*d.*, for the interest required. (See ART. **93**.)

12. What is the interest of £14 5*s.* $3\frac{1}{2}$*d.*, for 4 years 6 months and 14 days, at 7 per cent.?

Ans. £4 10*s.* $7\frac{72}{100}$*d.* nearly.

13. What is the interest of £1 7*s.* 6*d.*, for 2 years and 6 months, at $4\frac{1}{2}$ per cent.? *Ans.* £0 3*s.* $1\frac{1}{8}$*d.*

14. What is the interest of £105 10*s.* 6*d.*, for $9\frac{1}{2}$ months, at 5 per cent.? *Ans.* £4 3*s.* 6*d.* 1.95*far.*

PARTIAL PAYMENTS.

103. When Notes, bonds, or obligations, receive partial payments, or endorsements, we must use the following rule, which was given by CHANCELLOR KENT, in the New York Chancery Report:

RULE.

"*The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceed the interest, the surplus goes towards discharging the*

ncipal, and the subsequent interest is to be computed on balance of principal remaining due. If the payment be s than the interest, the surplus of interest must not be :en to augment the principal; but interest continues on the rmer principal until the period when the payments taken :ether, exceed the interest due, and then the surplus is to applied towards discharging the principal; and interest is be computed on the balance, as aforesaid.

By whom was the above rule given? You first compute the inest on the note from the time it was given up to what time? In ie the payment is greater than the interest, how do you proceed? t suppose the payment is less than the interest, how then do you ? Repeat the entire Rule.

EXAMPLES.

UTICA, Nov. 1, 1837.

1. For value received, I promise to pay Thomas Jones, order, the sum of six hundred and twenty dollars, on mand, with interest. CHARLES BANK.

The following endorsements were made on this note:

1838, Oct. 6,	there	was	endorsed		$61.07
1839, March 4,	"	"	"		89.03
1839, Dec. 11,	"	"	"		107.77
1840, July 20,	"	"	"		200.50

What was the balance due, Oct. 15, 1840, allowing 7 r cent. interest?

The student will find it convenient to arrange the work : finding the *multipliers* at 6 per cent. as follows:

	year.	*mo.*	*da.*	*mo.*	*da.*	*multipliers at 6 p. c.*
ate of note	1837	10	1			
t endorsement	1838	9	6	11	5	$0.055\frac{5}{6}$
endorsement	1839	2	4	4	28	$0.024\frac{2}{3}$
endorsement	1839	11	11	9	7	$0.046\frac{1}{6}$
h endorsement	1840	6	20	7	9	0.0365
ate of settlement	1840	9	15	2	25	$0.014\frac{1}{6}$

Having found the *multipliers*, we continue the work as llows:

The amount of note, or principal, is	$620.000
Interest on the same, up to Oct. 6, 1838, is	40.386
Amount due on note, Oct. 6, 1838, is	660.386
The first endorsement is	61.070
	599.316
Interest from Oct. 6, 1838, to March 4, 1839, is	17.247
Amount due March 4, 1839, is	616.563
The second endorsement is	89.030
	527.533
Interest from March 4, 1839, to Dec. 11, 1839, is	28.414
	555.947
The third endorsement is	107.770
	448.177
Interest from Dec. 11, 1839, to July 20, 1840, is	19.085
	467.262
The fourth endorsement is	200.500
	266.762
Interest from July 20, 1840, to Oct. 15, 1840, is	4.409
Ans.	271.171

UTICA, May 1, 1836.

2. For value received, I promise to pay Isaac Clark, or order, three hundred and forty-nine dollars ninety-nine cents and eight mills, with interest, at 6 per cent.

N. BROWN.

Endorsements were made on this note as follows:

Dec. 25, 1836,	there was paid	$49.998
June 30, 1837,	" " "	4.998
Aug. 22, 1838,	" " "	15.000
June 4, 1839,	" " "	99.999

How much was due April 5, 1840?

	year.	mo.	da.	yr.	mo.	da.	multipliers at 6 per cent.
Date of note	1836	4	1				
First endorsement	1836	11	25		7	24	\$0.039
Second "	1837	5	30		6	5	$0.030\frac{5}{6}$
Third "	1838	7	22	1	1	22	$0.068\frac{2}{3}$
Fourth "	1839	5	4		9	12	0.047
Date of settlement	1840	3	5		10	1	$0.050\frac{1}{6}$

The amount of the note, or principal, is		\$349.998
Interest up to Dec. 25, 1836, is		13.650
		363.648
The first endorsement is		49.998
		313.650
Interest up to June 4, 1839, is		45.950
		359.600
Endorsement June 30, 1837, which is less than the interest then due,	\$4.998	
Endorsement Aug. 22, 1838,	15.000	
	19.998	
This sum is still less than the interest now due.		
Endorsement June 4, 1839,	99.999	
		\$119.997
This sum exceeds the interest now due.		
		239.603
Interest up to April 5, 1840, is		12.020
Amount due April 5, 1840,		*Ans.* 251.623

UTICA, Dec. 9, 1835.

3. For value received, I promise to pay Peter Smith, or order, one hundred and eight dollars and forty-three cents, on demand, with interest, at 7 per cent.

JOHN SAVEALL.

Endorsements were made as follows:

March 3, 1836, there was endorsed				$50.04
Dec. 10, 1836,	"	"	"	13.19
May 1, 1838,	"	"	"	50.11

How much remained due Oct. 9, 1840?

Let the student find these multipliers for himself.

Multipliers at 6 *per cent.*

$0.014
$0.046\frac{1}{6}$
0.0835
$0.146\frac{1}{3}$

Ans. $5.844.

UTICA, Aug. 1, 1837.

4. For value received, I promise to pay D. Budlong, or bearer, one hundred and forty-three dollars and fifty cents, on demand, with interest. W. GOULD.

Dec. 17, 1837, there was endorsed				$37.40
July 1, 1838,	"	"	"	7.09
Dec. 22, 1839,	"	"	"	13.13
Sept. 9, 1840,	"	"	"	50.50

How much remains due Dec. 28, 1840, the interest being 7 per cent.?

Multipliers at 6 *per cent.*

$$0.022\frac{2}{3}
$0.032\frac{1}{3}$
0.0885
$0.042\frac{5}{6}$
$0.018\frac{1}{6}$

Ans. $60.866.

5. A note of $486 is dated Sept. 7, 1831, on which,

March 22, 1832, there was paid				$125
Nov. 29, 1832,	"	"	"	150
May 13, 1833,	"	"	"	120

What was the balance due April 19, 1834, the interest being 7 per cent.?

Multipliers at 6 per cent.

$0.0325
0.041$\frac{1}{6}$
0.027$\frac{1}{3}$
0.056

Ans. $144.404

104. The principal, the rate per cent., the time, and the interest, are so related to each other, that any three of them being given, the remaining one can be found.

PROBLEM I.

Given the principal, the rate per cent., and the time, to find the interest. The rule for this problem has already been given under Case III. ART. **102**; it is equivalent to the following

RULE.

Multiply the interest of $1, for the given time, and given rate per cent., by the principal.

PROBLEM II.

Given the time, the rate per cent., and the interest, to find the principal. By the reverse of the last problem, we obtain this

RULE.

Divide the given interest by the interest of $1, for the given time, and given rate per cent.

EXAMPLES.

1. The interest on a certain principal, for 9 months and 10 days, at 4½ per cent., is $1.01605. What was the principal?

In this example, we find the interest of $1, for 9 months

10 days, at $4\frac{1}{2}$ per cent., to be \$0.035; therefore, \$1.01605, divided by \$0.035, gives \$29.03, for the principal required.

2. What principal will, in 1 year 7 months and 15 days, at 6 per cent., give \$9.75 interest? *Ans.* \$100.

3. What principal will, in 7 years and 9 days, at 6 per cent., give \$16.86 interest? *Ans.* \$40.

4. What principal will, in 3 years and 6 months, at 5 per cent., give \$92.75 interest? *Ans.* \$530.

5. What principal will, in 3 months and 9 days, at 8 per cent., give \$90 interest? *Ans.* \$4090.909.

PROBLEM III.

Given the principal, the time, and the interest, to find the rate per cent.

RULE.

Divide the given interest by the interest of the given principal, for the given time, at 1 per cent.

EXAMPLES.

1. The interest of \$100, for 9 months and 10 days, is \$3.50. What is the rate per cent.?

In this example, we find the interest of \$100, for 9 months and 10 days, at 6 per cent., to be \$4.66$\frac{2}{3}$. The interest at 1 per cent. is \$0.77$\frac{7}{9}$; therefore, dividing \$3.50 by \$0.77$\frac{7}{9}$, we obtain $4\frac{1}{2}$ for the rate per cent. required.

2. At what rate per cent. will \$530, in 3 years and 6 months, give \$92.75 interest? *Ans.* 5 per cent.

3. At what rate per cent. will \$19.41, in 1 year 7 months and 13 days, give \$2.200339$\frac{1}{6}$ interest? *Ans.* 7 per cent.

4. At what rate per cent. will \$5.37, in 4 years and 12 days, give \$1.73272 interest? *Ans.* 8 per cent.

5. At what rate per cent. will \$4070, in 3 months, give \$91.575 interest? *Ans.* 9 per cent.

PROBLEM IV.

Given the principal, the rate per cent., and the interest, to find the time.

RULE.

Divide the given interest by the interest of the given principal, for 1 year, at the given rate per cent.

EXAMPLES.

1. In what time will \$37.13, at $4\frac{1}{2}$ per cent., give \$0.7518825 interest?

In this example, we find the interest of \$37.13, for 1 year, at $4\frac{1}{2}$ per cent., to be \$1.67085; therefore, dividing \$0.7518825 by \$1.67085, we get 0.45 years; this reduced to months and days, gives 5 months and 12 days.

2. In what time will \$700, at 7 per cent., give \$85.75 interest? *Ans.* 1 year 9 months.

3. In what time will \$100, at 6 per cent., give \$100 interest? That is, in what time will a given principal double itself at 6 per cent. interest? *Ans.* $16\frac{2}{3}$ years.

4. In what time will a given principal double itself at 7 per cent.? *Ans.* $14\frac{2}{7}$ years.

5. In what time will a given principal double itself at 8 per cent.? *Ans.* $12\frac{1}{2}$ years.

6. In what time will a given principal double itself at 5 per cent.? *Ans.* 20 years.

7. In what time will a given principal double itself at $4\frac{1}{2}$ per cent.? *Ans.* $22\frac{2}{9}$ years.

The following table gives the time required for a given principal to double itself at simple interest.

Per cent.	Years.	Per cent.	Years.	Per cent.	Years.
1	100	4	25	7	$14\frac{2}{7}$
$1\frac{1}{2}$	$66\frac{2}{3}$	$4\frac{1}{2}$	$22\frac{2}{9}$	$7\frac{1}{2}$	$13\frac{1}{3}$
2	50	5	20	8	$12\frac{1}{2}$
$2\frac{1}{2}$	40	$5\frac{1}{2}$	$18\frac{2}{11}$	$8\frac{1}{2}$	$11\frac{13}{17}$
3	$33\frac{1}{3}$	6	$16\frac{2}{3}$	9	$11\frac{1}{9}$
$3\frac{1}{2}$	$28\frac{4}{7}$	$6\frac{1}{2}$	$15\frac{5}{13}$	$9\frac{1}{2}$	$10\frac{10}{19}$

DISCOUNT.

105. DISCOUNT is an allowance made for the payment of money before it is due.

The *present worth* of a debt, payable at some future time, without interest, is such a sum of money as will, in the given time, amount to the debt.

When the interest is at 6 per cent., the amount of $1, for one year, is $1.06; therefore, the present worth of $1.06, due one year hence, is $1. We may also infer that the present worth of any sum for one year, will be as many dollars as $1.06 is contained in the given sum. Hence, we have the following

RULE.

Find the amount of $1, for the given time, at the given rate per cent., then divide the sum by this amount, and it will give the present worth. Subtract the present worth from the amount, and it will give the discount.

What is Discount? What is the present worth of a sum of money due at some future period? What is the present worth of $1.06, due one year hence, at 6 per cent. interest? Repeat the Rule for computing discount.

EXAMPLES.

1. What is the present worth of $622.75, due 3 years and 6 months hence, at 5 per cent.?

In this example, we find the amount of $1, for 3 years and 6 months, at 5 per cent., to be $1.175; therefore, dividing $622.75 by $1.175, we get $530, for the present worth. If we subtract the present worth from the sum, we get $92.75 for the discount.

2. What is the present worth of $4161.575, due 3 months hence, at 9 per cent.? *Ans.* $4070.

3. What is the present worth of $7.10272, due 4 years and 12 days hence, at 8 per cent.? *Ans.* $5.37.

4. Sold goods for $1500, to be paid one half in 6 months, and the other half in 9 months. What is the present worth of the goods, interest being at 7 per cent.? *Ans.* $1437.227.

5. Sold goods for $1500, to be paid at the end of 7½ months. What is the present worth of the goods, interest being at 7 per cent.? *Ans*. $1437.126.

NOTE.—We see that in this case I do not receive quite as much for my goods as I did in the case of example 4, provided I consider the present worth.*

6. What is the present worth of $50, payable at the end of 3 months, at 7 per cent.? *Ans*. $49.14.

7. What is the discount on $100, due 6 months hence, at 6 per cent.? *Ans*. $2.913.

8. What is the discount on $750, due 9 months hence, at 7 per cent.? *Ans*. $37.411.

9. What is the present worth of $3471.20, due 3 years and 9 months hence, at 4½ per cent.? *Ans*. $2970.011.

10. What is the discount of $150, due 3 months and 18 days hence, at 6 per cent.? *Ans*. $2.652.

11. What is the discount of $961.13, due 1 year and 5 months hence, at 7 per cent.? *Ans*. $86.713.

12. What is the discount of $37.40, due at the end of 7 months, at 6 per cent.? *Ans*. $1.265.

COMPOUND INTEREST.

106. When at the end of each year, the interest due is added to the principal, and the amount thus obtained is considered as a new principal, upon which the interest is cast for another year, and added to it to form a new principal for the next year, and so on to the last year, the last amount thus obtained, is called the AMOUNT AT COMPOUND INTEREST. If from this amount we subtract the original principal, we obtain the COMPOUND INTEREST.

How is the amount of compound interest found? How is the compound interest obtained?

* For still further developments of this curious subject, see Higher Arithmetic, ART. 67.

EXAMPLES.

1. What is the compound interest of $1000, for 3 years at 7 per cent.?

Principal,	$1000
Interest on $1000 for one year,	70
First amount, or second principal,	1070
Interest on $1070 for one year,	74.90
Second amount, or third principal,	1144.90
Interest on $1144.90 for one year,	80.143
Third amount,	1225.043
Original principal,	1000
The compound interest required,	*Ans.* $225.043

2. What is the compound interest of $100, for 4 years, at 6 per cent.?

Principal,	$100
Interest for first year,	6
First amount, or second principal,	106
Interest for second year,	6.36
Second amount, or third principal,	112.36
Interest for third year,	6.74
Third amount, or fourth principal,	119.10
Interest for fourth year,	7.15
Fourth amount,	126.25
Original principal,	100.
Compound interest required,	*Ans.* $26.25

3. What is the amount of $100, at 6 per cent. per annum, compound interest, for 2 years, when the interest is added in at the end of every six months?

Principal,	$100
Interest on $100 for 6 months, at 6 per cent.,	3
	103
Interest on $103 for 6 months,	3.09
	106.09
Interest on $106.09 for 6 months,	3.1827
	109.2727
Interest on $109.2727 for 6 months,	3.278181
Amount required,	$112.550881

4. What is the compound interest of $630, for 4 years, at 5 per cent.? *Ans.* $135.769.

5. What is the compound interest of $50, for 3 years, at 5 per cent.? *Ans.* $7.881.

6. What is the compound interest of $1000, for 4 years, at 6 per cent.? *Ans.* $262.477.

BANKING.

107. A BANK is an incorporated institution, created for the purpose of loaning money, receiving deposites, and dealing in exchange.

The *stock*, or amount of money in trade, is limited by law, and owned by various individuals, who are called *stockholders*.

Banks are allowed to make notes, which are denominated *bank bills*, which circulate as money, because they are obliged to redeem them with *specie*.

It is customary for banks, in most cases, when they

loan money, to take the interest in advance;* that is, to deduct it from the face of the note, at the time the money is lent. The note is then said to be *discounted*.

The sum to be discounted, or the face of the note, is called the *amount*.

The interest deducted is called the *discount*.

What then remains is called the *present worth*, or *proceeds*.

A note to be discounted, or bankable, must be made payable at some future time, and to the order of some person who endorses it.

It is usual for the banks to take the interest for 3 days more than the time specified in the note; and the borrower is not obliged to make payment till these three days have expired, which are for this reason called *days of grace*.

To find the banking discount on any sum of money, we have this

RULE.

Compute the interest by Case III. ART. **102**, *on the given sum, for three days more than is specified.*

What is a bank? What is the stock? Who are the stockholders? How are bank notes called? Do they circulate as money? How are the banks obliged to redeem their notes? How do banks sometimes take the interest? When is a note said to be discounted? What is the amount? What is the interest deducted called? How is that which remains called? Does a bank note require an endorser? For how many days more than specified in the note do banks take interest? What are these three days called?

EXAMPLES.

1. What is the banking discount on $1000, for 3 months, at 7 per cent.?

In this example, we find the interest on $1, for 3 months and 3 days, at 6 per cent., to be $0.0155, which,

* This method of discounting bank notes is usurious, and is fast going out of use, and instead of it the banks now deduct the discount as found by Rule under ART. 105.

multiplied by $1000, gives $15.50, for the discount at 6 per cent.; this, increased by its sixth part, becomes 18.08\frac{1}{3}$ for the discount at 7 per cent., as required.

2. What is the banking discount of $150, for 6 months, at 6 per cent.? *Ans.* $4.575.

3. What is the banking discount of $375, for 3 months and 9 days, at 7 per cent.? *Ans.* $7.438.

4. What is the banking discount of $400, for 9 months, at 7 per cent.? *Ans.* 21.23\frac{1}{3}$.

5. What is the banking discount of $29.30, for 7 months, at 5 per cent.? *Ans.* $0.867.

6. What is the banking discount of $472, for 10 months, at 7 per cent.? *Ans.* $27.809.

When the present worth of a bankable note, the time for which it is to be discounted, and the rate per cent. is given to find the amount, we have this

RULE.

Compute the banking discount on $1, *for the given time and rate per cent., subtract this discount from* $1, *then divide the present worth by the remainder, and the quotient will be the amount.*

EXAMPLES.

1. What must be the amount of a bankable note, so that when discounted for 3 months, at 6 per cent., it shall give a present worth of $600?

In this example, we find the banking discount on $1, for 3 months, to be $0.0155, which, subtracted from $1, gives $0.9845; therefore, dividing $600 by $0.9845, we obtain $609.446, for the required amount of the note.

2. What must be the face of a bankable note, so that when discounted for 2 months, at 7 per cent., the borrower shall receive $50? *Ans.* $50.62.

The following table gives the amount of a bankable note, so that when discounted at 5, 6, or 7 per cent., for any number of months, from 1 to 12, the present worth shall be just $1.

Months.	5 per cent.	6 per cent.	7 per cent.
1	1.004604	1.005530	1.006458
2	1.008827	1.010611	1.012402
3	1.013085	1.015744	1.018416
4	1.017380	1.020929	1.024503
5	1.021711	1.026167	1.030662
6	1.026079	1.031460	1.036896
7	1.030485	1.036807	1.043206
8	1.034929	1.042095	1.049593
9	1.039411	1.047669	1.056059
10	1.043932	1.053186	1.062605
11	1.048493	1.058761	1.069233
12	1.053093	1.064396	1.075944

We will now work some examples by the aid of the above table.

3. What must be the face of a bankable note, so that when discounted for 10 months at 5 per cent., the present worth may be $1000?

Looking in the table directly under the 5 per cent., and adjacent to 10 months, we find $1.043932, this, multiplied by $1000, gives $1043.932, for the face of the note required.

4. What must be the face of a bankable note, so that when discounted for 7 months, at 7 per cent., the present worth may be $70.50? *Ans.* $73.546.

5. What amount must I make my note, so that when discounted at the bank for 12 months, at 7 per cent., I may receive $100? *Ans.* $107.594.

6. What must be the amount of a note, so that when discounted at the bank for 6 months, at 6 per cent., the borrower may receive $365? *Ans.* $376.483.

7. What must be the amount of a note, so that when discounted at the bank for 9 months, at 7 per cent., the borrower may receive $500? *Ans.* $528.03.

PERCENTAGE.

108. The term *per cent.* is an abbreviation of *per centum*, which means by the hundred.

Thus, 5 out of 100 is 5 per cent.
6 out of 100 is 6 per cent.
7 out of 100 is 7 per cent.

And so for other rates per cent.

Different rates per cent. are most conveniently expressed by means of decimals.

Thus, 1	per cent. is the	same as	0.01
2	"	"	0.02
3	"	"	0.03
4	"	"	0.04
5	"	"	0.05

Suppose we wish 5 per cent. of $1122, we must take $\frac{5}{100}$ of it; this is done by multiplying by the decimal 0.05.

OPERATION.

$$\begin{array}{r} \$1122 \\ 0.05 \\ \hline \textit{Ans.}\ \$56.10 \end{array}$$

Hence, to find the percentage of any number, we have this

RULE.

Multiply the given number by the percentage expressed in decimals.

From what is *per cent.* abbreviated? And what does it mean? 5 out of 100 is what per cent.? 6 out of 100 is what per cent.? 7 out of 100 is what per cent.? What is the decimal expression for 1 per cent.? What for 2, 3, 4, &c., per cent.? Repeat the Rule for finding the per cent. of a number.

EXAMPLES.

1. What is $4\frac{1}{2}$ per cent. of $10000?

$$\begin{array}{lr} \text{In this example } 4 \text{ per cent. is} & 0.04 \\ \tfrac{1}{2} \text{ per cent. is} & 0.005 \\ \hline 4\tfrac{1}{2} \text{ per cent. is} & 0.045 \end{array}$$

OPERATION.

$$\begin{array}{r} \$10000 \\ 0.045 \\ \hline 50000 \\ 40000 \\ \hline \textit{Ans.}\ \$450.000 \end{array}$$

2. What is 1 per cent. of $730? *Ans.* $7.30.

3. What is 3 per cent. of 5789 pounds? *Ans.* 173.67*lb.*

4. What is 4 per cent. of 365 bushels? *Ans.* 14.6*bu.*

5. What is $4\frac{1}{2}$ per cent. of $75.03? *Ans.* $3.37635.

6. What is 7 per cent. of 2345? *Ans.* 164.15.

7. What is 30 per cent. of $495? *Ans.* $148.50.

8. A person laid out $222 as follows: he gave 21 per cent. of his money for calicoes: 15 per cent. for thread: 45 per cent. for silks; and the remaining 19 per cent. for broadcloths. How many dollars did he expend for each?

Ans.
He gave for	calicoes,	$46.62
"	thread,	33.30
"	silks,	99.90
"	broadcloths,	42.18

9. A merchant having 500 barrels of flour, sold at one time 25 per cent. of it, at another time he sold 20 per cent. of the remainder. How many barrels did he sell at each time, and how many remain?

Ans.
The first time he sold 125 barrels.
The second time he sold 75 barrels.
He has remaining 300 barrels.

COMMISSION.

109. COMMISSION is an allowance made to a factor or commission merchant for buying and selling. It is estimated at so much per cent. on the money used in the transaction.

What is Commission? How is it estimated?

EXAMPLES.

1. What is the commission on \$3765.50, at $3\frac{1}{2}$ per cent.?

OPERATION.

$$\begin{array}{r} \$3765.50 \\ 0.035 \\ \hline 1882750 \\ 1129650 \\ \hline \textit{Ans.}\ \$131.79250 \end{array}$$

2. What is the commission on \$10000, at 4 per cent.? *Ans.* \$400.

3. A factor sells 43 bales of cotton at \$375 per bale, and charges 2 per cent. commission. How much money must he pay to his principal? \$15802.50.

4. A sends to B, a broker, \$3605 to be invested in stock: B is to receive 3 per cent. on the amount paid for the stock. What was the value of the stock purchased?

Since B is to receive 3 per cent., it is plain that \$103 of A's money would purchase \$100 worth of stock. Hence, the amount expended for stock must be $\frac{100}{103}$ of \$3605= \$3500. *Ans.*

Commission on \$3500 at 3 per cent., is \$105, which, added to \$3500 makes \$3605, which shows the work to be correct.

5. A factor receives \$60112, and is directed to purchase cotton at \$289 per bale: he is to receive 4 per cent. on the money paid for the cotton. How many bales did he purchase?

$\frac{100}{104}$ of \$60112 = \$57800 amount paid for cotton.

\$57800 ÷ \$289 = 200 number of bales.

6. The par value, or first cost of 125 shares of bank stock, was \$100 per share. What is the present value, if the stock is worth 18 per cent. above par? *Ans.* \$14750.

7. What is the value of 50 shares of bank stock, the par value of which was \$200 per share, on the supposition that it is 12 per cent. below par, or, that it is worth only 88 per cent. of its par value? *Ans.* \$8800.

8. A bank fails and has in circulation \$108567. It can pay only 13 per cent. What amount of money has it on hand? *Ans.* \$14113.71.

INSURANCE.

110. INSURANCE is a contract, by which an individual or company, agrees to restore the value of ships, houses, and goods, of whatever kind, which may be destroyed by the perils of the sea, or by fire.

The security is given in consideration of a certain sum of money called the *premium*, which is paid by the owner of the property insured.

The premium is always estimated at a certain rate per cent. on the value of the property insured.

The written agreement of indemnity, is called a *policy*.

What is Insurance? What is premium? How is the premium estimated? What is the policy?

EXAMPLES.

1. If A gets his house insured for \$1800, at 41 cents on \$100, what will be the amount of the premium?

Ans. \$7.38.

2. An insurance of $12000 was effected on the ship Ocean, at a premium of 2 per cent. What did the premium amount to? *Ans.* $240.

3. I effected an insurance of $5230 on my dwelling-house and furniture for 1 year, at $\frac{3}{8}$ of 1 per cent. What did the premium amount to? *Ans.* $19.6125.

4. What is the amount of premium for insuring $34567, at 60 cents on $100? *Ans.* $207.402.

5. What would be the premium for insuring a ship and cargo, valued at $46370, from Boston to Liverpool, at $2\frac{1}{4}$ per cent.? *Ans.* $1043.325.

LOSS AND GAIN

111. Loss and Gain is a rule by which merchants discover the amount lost or gained in buying and selling goods. It also assists them in adjusting the price of their goods so as to lose or gain a certain per cent.

What is Loss and Gain?

EXAMPLES.

1. Bought 300 yards of broadcloth at $2.25 per yard, and sold the same at $3.50 per yard. How much was gained?

$3.50 price of 1 yard.
$2.25 cost of 1 yard.
$1.25 gain on 1 yard.

$1.25
300
Ans. $375.00 whole gain.

2. Bought 75 pounds of coffee at 10 cents per pound. At how much per pound must I sell it so as to gain $3 on whole?

75 pounds, at 10 cents per pound is $7.50
Gain $3.00
$10.50

Hence, I must sell the 75 pounds for $10.50.

Therefore, $10.50÷75=14 cents per pound, for the price at which I must sell it.

3. A merchant bought 320 barrels of flour for $5 per barrel, but he finds he must lose 10 per cent. in the sales. How much will he receive for the whole?

320 barrels.
$5
$1600 whole cost.

Since he loses 10 per cent., one dollar's worth must sell for 90 cents.

$1600
0.90
Ans. 1440.00 what he receives.

4. Suppose I buy 25 cords of maple wood at $2.50 per cord, and sell it so as to make 25 per cent. What must I receive for the whole?

$2.50 cost of 1 cord.
25
1250
500
$62.50 cost of 25 cords.

Since I make 25 per cent., one dollar's worth must sell for $1.25.

$62.50
1.25
31250
12500
6250
Ans. $78.1250 what I receive.

5. Bought a house and lot for \$1400 and sold it for \$1200. How much per cent. did I lose?

\$1400 cost of house.
\$1200 what sold for.
\$200 what I lost on \$1400.

Hence, $\frac{200}{1400}=\frac{1}{7}=0.14\frac{2}{7}=14\frac{2}{7}$ per cent.

6. Bought 25 hogsheads of molasses, at \$18 per hogshead, in Havana; paid duties, \$16.30; freight, \$25; cartage, \$5.50; insurance, \$25.25. What per cent. shall I gain if I sell it at \$28 per hogshead?

\$700 what I received for 25*hhd.* at \$28 per *hhd.*
\$522.05 whole cost of 25 hogsheads.
\$177.95 gain on \$522.05.

$$\frac{177.95}{522.02}=0.34+\text{about } 34 \text{ per cent.}$$

7. Bought 224 gallons of molasses for 26 cents per gallon, and sold the whole for \$64.064. What did I gain per cent.?

224 number of gallons.
\$0.26 cost of 1 gallon.
1344
448
\$58.24 whole cost.

\$64.064 amount sold for.
\$58.24
\$5.824 gain on \$58.24.

Hence, $\frac{5.824}{58.24}=0.10=10$ per cent.

FELLOWSHIP.

112. FELLOWSHIP is the union of two or more individuals in trade, with an agreement to share the losses and profits in the ratio of the amount which each individual puts into the partnership. The money employed is called the *capital stock*.

The loss or gain to be shared is called the *dividend.*

What is Fellowship? What is the capital stock? What is the dividend?

EXAMPLES.

1. A, B, and C, enter into partnership. A put in \$180, B put in \$240, and C put in \$480. They gained \$300. What is each one's part of the gain?

\$180	A's stock.
240	B's stock.
480	C's stock.
\$900	whole stock.

$\frac{180}{900}=\frac{1}{5}$ A's part of the entire stock.
$\frac{240}{900}=\frac{4}{15}$ B's " " "
$\frac{480}{900}=\frac{8}{15}$ C's " " "

Hence, A must have	$\frac{1}{5}$	of \$300=	\$60	for his gain
B "	$\frac{4}{15}$	of \$300=	\$80	"
C "	$\frac{8}{15}$	of \$300=	\$160	"
			\$300	proof.

2. Five persons, A, B, C, D, and E, are to share between them \$2400. A is to have $\frac{1}{6}$; B is to have $\frac{1}{4}$; C is to have $\frac{3}{8}$; D and E are to divide the remainder in proportion to the numbers 5 and 7. How much does each one receive?

A receives	$\frac{1}{6}$ of	\$2400=	\$400
B "	$\frac{1}{4}$ of	2400=	600
C "	$\frac{3}{8}$ of	2400=	900
			\$1900

$$
\begin{array}{r}
\$2400 \\
1900 \\
\hline
500
\end{array}
\text{ sum of D's and E's part.}
$$

5 represents D's part.
7 represents E's part.
12

Hence, D must receive $\frac{5}{12}$ of $500=$208.33$\frac{1}{3}$.
E must receive $\frac{7}{12}$ of 500= 291.66$\frac{2}{3}$.

3. There are three horses belonging to three men, employed to draw a load of plaster a certain distance for $26.45. It is estimated that A's and B's horses do $\frac{3}{4}$ of the labor; A's and C's horses $\frac{9}{10}$; B's and C's horses $\frac{13}{20}$. They are to be paid proportionally according to these estimates. What ought each man to receive?

A's and B's horses do $\frac{3}{4}=\frac{15}{20}$
A's and C's horses do $\frac{9}{10}=\frac{18}{20}$
B's and C's horses do $\frac{13}{20}=\frac{13}{20}$

Adding all these fractions together, we shall obtain twice what they all do, according to the above estimate; if we take half the sum, it will give the part supposed to be done by all.

Hence, A's, B's, and C's horses do $\frac{23}{20}$.

If from this fraction we subtract $\frac{13}{20}$, which is B's and C's, we find $\frac{10}{20}$ for the part supposed to be done by A's horse. In the same way we find $\frac{5}{20}$ for the part done by B's horse. $\frac{8}{20}$ will represent the part done by C's horse.

Therefore, the parts which the three horses are supposed to do are $\frac{10}{20}$, $\frac{5}{20}$, $\frac{8}{20}$. These fractions, having a common denominator, must be to each other as their numerators, that is, as 10, 5, 8, whose sum is 23.

Hence, A ought to have $\frac{10}{23}$ of $26.45=$11.50
B ought to have $\frac{5}{23}$ of 26.45= 5.75
C ought to have $\frac{8}{23}$ of 26.45= 9.20
Proof $26.45

4. A, B, and C, agree to contribute $365 towards building a church, which is to be at the distance of 2 miles

from A, $2\frac{7}{8}$ miles from B, and $3\frac{1}{2}$ from C. They agree that their shares shall be reciprocally proportional to their distances from the church. What ought each to contribute ?

The inverse ratio of numbers is found by taking the direct ratios of their reciprocals. The reciprocals of the numbers 2, $2\frac{7}{8}$, $3\frac{1}{2}$, are $\frac{1}{2}$, $\frac{8}{23}$, $\frac{2}{7}$; these reduced to a common denominator, become $\frac{161}{322}$, $\frac{112}{322}$, $\frac{92}{322}$. Now, we must obviously divide $365 in the proportion of these numerators; their sum is 365.

Hence, A must contribute	$\frac{161}{365}$ of $365	=	$161
B "	$\frac{112}{365}$ of 365	=	112
C "	$\frac{92}{365}$ of 365	=	92
		Proof	$365

5. A person wills to his two sons and a daughter, the following sums: To the elder son $1200, to the younger son $1000, and to his daughter $600; but it is found that his whole estate amounts to only $800. How much did each child receive ?

Ans. The elder son received 342.857\frac{1}{7}$.
The younger son received 285.71[illegible]
The daughter received 171.42[illegible]

DOUBLE FELLOWSHIP.

113. When the stock of the several partners continues in trade for unequal periods of time, the profit or loss must be apportioned with reference both to the stock and time. In such cases the fellowship is called DOUBLE FELLOWSHIP.

What is Double Fellowship ?

EXAMPLES.

1. Three partners, A, B, and C, put into trade money as follows: A put in $400 for 2 months: B put in $300 for

4 months; C put in $500 for 3 months. They gained $350. How must they share of this gain?

It is evident that $400 for 2 months is the same as $\$400\times2=\$800$ for one month.

And $300 for 4 months is the same as $\$300\times4=\$1200$ for one month.

And $500 for 3 months is the same as $\$500\times3=\$1500$ for one month.

Hence,	$800	A's money for one month.
	1200	B's money for one month.
	1500	C's money for one month.
	$3500	

Therefore, by Single Fellowship,

A must have $\frac{800}{3500}=\frac{8}{35}$ of $\$350=\$80$
B " " $\frac{1200}{3500}=\frac{12}{35}$ of $350=120$
C " " $\frac{1500}{3500}=\frac{3}{7}$ of $350=150$
$350 Proof.

RULE.

Multiply each partner's stock by the time it was in trade; make each product the numerator of a fraction, and the sum of the products a common denominator; then multiply the whole gain or loss by each of these fractions, for each partner's share.

Repeat this Rule.

2. Three farmers hired a pasture for $55.50 for the season. A put in 6 cows for 3 months, B put in 8 cows for 2 months, C put in 10 cows for 4 months. What rent ought each to pay?

Ans. A ought to pay $13.50. B " 12.00. C " 30.00.

3. On the first day of January, A began business with $650; on the first day of April following, he took B into partnership with $500; on the first day of next July, they

took in C with $450; at the end of the year they found they had gained $375. What share of the gain had each?

$$\textit{Ans.}\left\{\begin{array}{lr} \text{A had} & \$195. \\ \text{B } `` & 112.50. \\ \text{C } `` & 67.50. \end{array}\right.$$

4. A, B, and C, have together performed a piece of work for which they receive $94. A worked 12 days of 10 hours each; B worked 15 days of 6 hours each; C worked 9 days of 8 hours each. How ought the $94 to be divided between them?

$$\begin{array}{llrl} \text{A} & \text{worked } 12\times10= & 120 & \text{hours.} \\ \text{B} & \quad `` \quad 15\times\ 6= & 90 & \text{hours.} \\ \text{C} & \quad `` \quad\ \ 9\times\ 8= & \underline{72} & \text{hours.} \\ & & 282 & \end{array}$$

$$\begin{array}{l} \text{Therefore, A had } \tfrac{120}{282} \text{ of } \$94=\tfrac{20}{47} \text{ of } \$94=\$40. \\ \qquad\qquad\ \ \text{B had } \tfrac{90}{282} \text{ of } \ 94=\tfrac{15}{47} \text{ of } \ 94=\ 30. \\ \qquad\qquad\ \ \text{C had } \tfrac{72}{282} \text{ of } \ 94=\tfrac{12}{47} \text{ of } \ 94=\ 24. \end{array}$$

5. A ship's company take a prize of $4440, which they agree to divide among them according to their pay and the time they have been on board. Now the officers and midshipmen have been on board 6 months, and the sailors 3 months; the officers have $12 per month, the midshipmen $8, and the sailors $6 per month; moreover, there are 4 officers, 12 midshipmen, and 100 sailors. What will each one's share be?

$$\textit{Ans.}\left\{\begin{array}{lr} \text{Each officer must have} & \$120. \\ \text{Each midshipman } `` & 80. \\ \text{Each sailor } `` & 30 \end{array}\right.$$

ASSESSMENT OF TAXES.

114. TAXES are moneys paid by the people for the support of government. They are assessed on the citizens in proportion to their property; except the *poll tax*, which is so much for each individual, without regard to his property.

In order to ascertain what each individual ought to pay, an accurate inventory of all the taxable property must be made.

When a tax is to be assessed on property and polls, we must first see how much the polls amount to, which must be deducted from the whole sum to be raised; we must then apportion the remainder according to each individual's property.

To effect this apportionment, we find what per cent. of the whole property to be taxed, the sum to be raised is; we then multiply each one's inventory by this per cent., expressed in decimals, and the product is his tax.

Assessors find it convenient to form a table which shall at once give the taxes on small sums from one dollar and upwards.

What are taxes? How are they assessed? What is a poll tax? Why must [illegible] inventory of all the taxable property be made? W[illegible] tax is to be laid on property and polls, which [illegible] first? Having deducted the amount of poll taxes [illegible] Having found the tax on one dollar, [illegible] any other amount? May the labor [illegible]

[illegible]AMPLES.

[illegible] $600 is t[illegible]wn con[illegible] the whol[illegible] amounts [illegible] poll [illegible] will be A's [illegible]y is ass[illegible] who pays

$0.75 amount of one poll tax.
60

$45.00 amount of all the poll taxes.

Deduct $600 whole amount to be raised.
45 amount of poll taxes.

$555 amount to be raised on $37000.

Hence, $\frac{555}{37000}$=$0.015 tax on one dollar.

Having found the tax on one dollar, we readily construct this

TABLE.

$1	pays	$0.015	$20	pays	$0.30	$200	pays	$3.00
2	"	.03	30	"	.45	300	"	4.50
3	"	.045	40	"	.60	400	"	6.00
4	"	.06	50	"	.75	500	"	7.50
5	"	.075	60	"	.90	600	"	9.00
6	"	.09	70	"	1.05	700	"	10.50
7	"	.105	80	"	1.20	800	"	12.00
8	"	.12	90	"	1.35	900	"	13.50
9	"	.135	100	"	1.50	1000	"	15.00
10	"	.15						

Now, to find A's tax, his property being $653, I find by the above Table, that

The tax on $600 is $9.00
The tax on 50 is .75
The tax on 3 is .045

The tax on $653 is $9.795
One poll is .75

$10.545 tax required.

By the ... what would be the tax on $425,
Ans. $6.375.

... what must B pay, who has 2 ... personal property is assessed at ...
Ans. $12.93.

EQUATION OF PAYMENTS.

115. Equation of Payments is a process by which we ascertain the average time for the payment of several sums due at different times.

What is Equation of Payments?

Suppose I owe \$1000, of which \$100 is due in 2 months, \$250 in 4 months, \$350 in 6 months, and \$300 in 9 months. Now, if I pay the whole sum at once, how many months credit ought I to have?

A credit on \$100 for 2 months is the same as a credit on \$1 for 200 months.	$\$100 \times 2mo. = 200mo.$
A credit on \$250 for 4 months is the same as a credit on \$1 for 1000 months.	$\$250 \times 4mo. = 1000mo.$
A credit on \$350 for 6 months is the same as a credit on \$1 for 2100 months.	$\$350 \times 6mo. = 2100mo.$
A credit on \$300 for 9 months is the same as a credit on \$1 for 2700 months.	$\$300 \times 9mo. = 2700mo.$
	\$1000 6000*mo.*

Hence, I ought to have the same as a credit on \$1 for 6000 months. But if I wish a credit on \$1000 instead of \$1, it ought evidently to be for only one thousandth part of 6000 months, which is 6 months.

Hence, we infer this

RULE.

Multiply each payment by the time before it becomes due; divide the sum of these products by the sum of the payments; the quotient will give the equated time.

EXAMPLES.

1. I purchased a bill of goods amounting to $1500, of which I am to pay $300 in 2 months, $500 in 4 months, and the balance in 6 months. What would be the mean time for the payment of the whole?

Ans. $4\frac{8}{15}$*mo.*, or 4*mo.* 16*da.*

2. A merchant owes $500 to be paid in 6 months, $600 to be paid in 8 months, and $400 to be paid in 12 months. What is the equated time of payment?

Ans. $8\frac{2}{5}$*mo.*, or 8 *mo.* 12*da.*

3. A owes B a certain sum; one third is due in 6 months, one fourth in 8 months, and the remainder in 12 months. What is the mean time of payment?

It is evident that it makes no difference what the amount is which A owes B, since it is certain fractional parts which become due at particular times. If we suppose the sum to be $1, then our work will be

$$\begin{array}{rl} \$ & mo. \quad mo. \\ \frac{1}{3} \times & 6 = 2 \\ \frac{1}{4} \times & 8 = 2 \\ \text{remainder is } \frac{5}{12}\text{, and } \frac{5}{12} \times & 12 = 5 \\ \hline & \textit{Ans. } 9 \text{ months.} \end{array}$$

The least sum which we can take so as to avoid fractions is $12. In this case we have

$$\begin{array}{lllll} \frac{1}{3} \text{ of } \$12 = & \$4, \text{ and} & \$4 \times & 6mo. = & 24mo. \\ \frac{1}{4} \text{ of } \quad 12 = & 3 \quad `` & 3 \times & 8mo. = & 24mo. \\ \text{remainder} \quad = & 5 \quad `` & 5 \times & 12mo. = & 60mo. \\ \hline & \$12 & & & 108mo. \end{array}$$

Hence, $\frac{108}{12} = 9$ months, for the time.

4. A merchant has due him $300 to be paid in 2 months; $800 to be paid in 5 months; $400 to be paid in 10 months. What is the equated time for the payment of the whole? *Ans.* $5\frac{11}{15}$*mo.*, or 5*mo.* 22*da.*

5. A merchant owes \$1200, payable as follows: \$200 in 2 months, \$400 in 5 months, and the remainder in 8 months. He wishes to pay the whole at one time. What is the equated time of such payment? *Ans.* 6 months.

6. A merchant bought goods to the amount of \$2400, for one fourth of which he was to pay cash at the time of receiving the goods, one third in 6 months, and the balance in 10 months. What was the equitable time for the payment of the whole?

$\frac{1}{4}$ of \$2400=\$600, which for 0 months gives	\$600 × 0=	0*mo.*
$\frac{1}{3}$ of \$2400=\$800, which for 6 months gives	\$800 × 6=	4800*mo.*
Balance=\$1000, which for 10 months gives	\$1000 × 10=	10000*mo.*
	\$2400	14800*mo.*

Hence, 14800*mo.* ÷ 2400 = $6\frac{1}{6}$ months for the time sought.

It is obvious that the time may be estimated in days as well as in months. To illustrate this we will give several examples of this kind.

7. Suppose I owe \$100 payable on January 1st, \$150 on February 5th, \$300 on April 10th. If we count from January 1st, and allow 29 days to February, it being Leap year, on what day ought the whole sum in equity to be paid?

Counting from January 1st, the \$100 will have no time of credit:	\$100 × 0*da.*=	0*da.*
From Jan. 1st to Feb. 5th is 35 days:	\$150 × 35*da.*=	5250*da.*
From Jan. 1st to April 10th is 100 days:	\$300 × 100*da.*=	30000*da.*
	\$550	35250*da.*

Hence, $35250da. \div 550 = 64\frac{1}{11}$ days, or counted from Jan. 1st, gives March 5th for the equated time of the payment of the whole.

8. A merchant bought a bill of goods amounting to \$1000. He agrees to pay \$250 the first day of the next March, \$250 on the 3d of the following May, \$250 on the 4th of the following July, and the remaining \$250 on the 15th of the following September. What would be the equitable time for paying the whole?

In this example, all the payments being equal, we may take for each one any sum we please. For simplicity we will consider each payment as \$1.

Counting from March 1st, we see that the first payment has no credit:	$\$1 \times 0$ days $=$	0 days.
From March 1st to May 3d is 63 days:	$\$1 \times 63$ days $=$	63 days.
From March 1st to July 4th is 125 days:	$\$1 \times 125$ days $=$	125 days.
From March 1st to Sept. 15th is 198 days:	$\$1 \times 198$ days $=$	198 days.
	\$4	386 days.

Hence, $386 \text{ days} \div 4 = 96\frac{1}{2}$ days. Calling this 97 days, and counting from March 1st, we have June 6th for the time sought.

When a debt due at some future period has received several partial payments before the time due, to find how long beyond this time the balance may in equity remain unpaid.

RULE.

Multiply each payment by the time it was paid before due; then divide the sum of these products by the balance remaining unpaid.

EXAMPLES.

9. Suppose \$1000 to be due at the end of 6 months, that 3 months before it is due there was paid \$100, and that 1

month before the expiration of the 6 months there was paid \$300. How long after the end of the 6 months may the balance of \$600 remain unpaid?

$$
\begin{aligned}
\$100 \times 3mo. &= \$300 \\
\$300 \times 1mo. &= \underline{\$300} \\
\$600)&\underline{\$600}
\end{aligned}
$$

Ans. 1 month.

10. Suppose \$1496.41 to be due at the end of 90 days, that 84 days before it is due there is paid \$500; 32 days before the 90 days expire there is paid \$502.50. How long after the 90 days before the balance of \$493.91 ought to be paid? *Ans.* 117½ days.

11. A lent \$200 to B for 8 months; at another time he lent him \$300 for 6 months. For how long a time ought B to lend A \$800 to balance the favor?

Ans. 4¼ months

INVOLUTION.

116. THE product arising from multiplying a number into itself is called the *second power*, or the *square* of that number. Thus, $3 \times 3 = 9$: the number 9 is the square of 3.

If the square of a number be again multiplied by that number, the result is called the *third power*, or the *cube* of the number. Thus, $3 \times 3 \times 3 = 27$: the number 27 is the cube of 3.

The word *power* denotes the product arising from multiplying a number into itself a certain number of times; and the number thus multiplied is called the *root*. Thus, 9 is the second power of 3, and 3 is the square root of 9. In the same manner 27 is the third power of 3, and 3 is the cube root of 27.

The product arising from multiplying a number into itself is called what? If it be used as a factor three times what power is it? The number 9 is what power of 3? The number 27 is what power of 3? What is the square root of 9? What is the cube root of 27?

117. *Involution is the method of finding the powers of numbers.*

To denote that a number is to be raised to a power, a small figure is placed above, a little to the right of the number whose power is to be found.

This small figure is called the *index*, or *exponent*.

Thus, $4^2=4\times4=16$; here the exponent is 2, and 4^2 denotes the second power of 4. In the same way we have

$3^1 = 3$ the first power of 3.

$3^2=3\times3= 9$ the second power of 3.

$3^3=3\times3\times3= 27$ the third power of 3.

$3^4=3\times3\times3\times3= 81$ the fourth power of 3.

$3^5=3\times3\times3\times3\times3=243$ the fifth power of 3

&c., &c.

Therefore, to raise a number to any power, we have the following

RULE.

Multiply the number continually by itself, as many times less one as there are units in the exponent; the last product will be the power sought.

What is Involution? How do we denote that a number is to be raised to a power? What is this small figure placed above, a little to the right, called? Repeat the Rule.

EXAMPLES.

1. What is the square of 23?

Ans. 529.

2. What is the cube of 17?

Ans. 4913.

3. What is the fifth power of 47?

Ans. 229345007.

4. What is the ninth power of 9?

Ans. 387420489

5. What is the square of 625?

Ans. 390625.

6. What is the cube of 48?

Ans. 110592.

7. What is the square of 0.75?

Ans. 0.5625.

8. What is the cube of 0.65?

Ans. 0.274625.

9. What is the square of $8\frac{1}{2}$?

Ans. $72\frac{1}{4}$.

10. What is the square of $\frac{3}{4}$?

Ans. $\frac{9}{16}$.

11. What is the cube of $\frac{7}{8}$?

Ans. $\frac{343}{512}$.

12. What is the cube power of $3\frac{1}{3}$?

Ans. $\frac{1000}{27}=37\frac{1}{27}$.

13. What is the fifth power of $2\frac{3}{4}$?

Ans. $\frac{161051}{1024}=157\frac{283}{1024}$.

14. What is the third power of 0.5?

Ans. 0.125.

15. What is the fourth power of 0.25?

Ans. 0.00390625.

16. What is the square of $\frac{4}{5}$?

Ans. $\frac{16}{25}$.

17. What is the cube of $1\frac{1}{2}$?

Ans. $3\frac{3}{8}$.

18. What is the cube of $2\frac{1}{5}$?

Ans. $10\frac{81}{125}$.

EVOLUTION.

118. EVOLUTION is the reverse of involution; that is, it explains the method of resolving a number into equal factors.

When a number can be resolved into a number of equal factors, such factor is called a *root*.

If the number is resolved into two equal factors, such factor is called the *square root*.

Thus, $36=6\times6$, and 6 is the square root of 36. In the same way 7 is the square root of 49, since $49=7\times7$.

To denote that the square root of a number is to be found, we use the symbol $\sqrt{}$. This sign is placed over the number whose square root is to be taken. Thus, $\sqrt{81}$ denotes that the square root of 81 is to be found; that is, $\sqrt{81}=9$; in the same way $\sqrt{100}=10$; $\sqrt{25}=5$; $\sqrt{36}=6$; $\sqrt{16}=4$.

When a number is resolved into three equal factors, such factor is called the *cube root*.

Thus, $64=4\times4\times4$, and 4 is the cube root of 64; in the same way 5 is the cube root of 125, since $125=5\times5\times5$.

To indicate that the cube root of a number is to be found, we use the symbol $\sqrt[3]{}$, placed over the number as in the case of the square root.

Thus, $\sqrt[3]{27}$ denotes that the cube root of 27 is to be found, that is, $\sqrt[3]{27}=3$; in the same way, $\sqrt[3]{64}=4$; $\sqrt[3]{8}=2$; $\sqrt[3]{216}=6$.

What is Evolution? When a number can be resolved into a number of equal factors, what is such a factor called? If the number is resolved into two equal factors, what is the root called? When resolved into three equal factors, what is the root called? What character is used to denote the square root? What to denote the cube root? What is the square root of 81? What is the square root of 100? What is the cube root of 27? What is the cube root of 8?

Before proceeding to explain the method of extracting the square roots of numbers, we will involve some numbers by considering them as decomposed into units, tens, hundreds, &c.

What is the square of 25? Of 35?

OPERATION.

$$\begin{array}{r} 25=20+\ 5 \\ 20+\ 5 \\ \hline 100+25 \\ 400+100 \\ \hline 25^2=400+200+25 \\ \hline \end{array}$$

OPERATION.

$$\begin{array}{r} 35=30+\ 5 \\ 30+\ 5 \\ \hline 150+25 \\ 900+150 \\ \hline 35^2=900+300+25 \\ \hline \end{array}$$

By a similar method, we find

$$\begin{aligned} 46^2&=1600+480+36 \\ 54^2&=2500+400+16 \\ 93^2&=8100+540+\ 9 \\ 48^2&=1600+640+64 \end{aligned}$$

If we wish to extract the square root of 1600+640+64, we proceed as follows:

We take the square root of 1600, which is 40; this is the first part of the root; its square being subtracted from 1600+640+64, leaves the remainder 640+64. We see that 640, divided by twice 40, or 80, gives 8 for a quotient, which is the second part of the root required.

CASE I.

From the above process, we deduce the following rule for the extraction of the square root of a whole number.

RULE.

I. Point off the given number into periods of two figures each, counting from the right toward the left. When the number of figures is odd, it is evident that the left-hand, or first period, will consist of but one figure.

II. Find the greatest square in the first period, and place its root at the right of the number, in the form of a quotient figure in division. Subtract the square of this root from the first period, and to the remainder annex the second period; the result will be the FIRST DIVIDEND.

III. Double the root already found, and place it on the left of the number, for the FIRST TRIAL DIVISOR. *See how many times this trial divisor, with a cipher annexed, is contained in the dividend, the quotient figure will be the second figure of the root; this must be placed at the right of the* TRIAL DIVISOR; *the result will be the* TRUE DIVISOR. *Multiply the true divisor by this second figure of the root, and subtract the product from the dividend, and to the remainder annex the next period, for a* SECOND DIVIDEND.

IV. To the last TRUE DIVISOR, *add the last figure of the root, for a new* TRIAL DIVISOR, *and continue to operate as before, until all the periods have been brought down.*

EXAMPLES.

1. What is the square root of 531441?

OPERATION.

		53′14′41(729	root.
	7	49	
First trial divisor,	14	414	1st dividend.
First true divisor,	142	284	
Second trial divisor,	144	13041	2d dividend.
Second true divisor,	1449	13041	
		0	

2. What is the square root of 11390625 ?

OPERATION.

3	11′39′06′25(3375
63	9
667	239
6745	189
	5006
	4669
	33725
	33725
	0

In the first example, we exhibited the trial divisors, as well as the true divisors, but in the second example we adhered more closely to our rule, and placed the succeeding figures of the root at the right of the trial divisors, without again writing them down.

3. What is the square root of 11019960576 ?

OPERATION.

1	1′10′19′96′05′76(104976
204	1
2089	1019
20987	816
209946	20396
	18801
	159505
	146909
	1259676
	1259676
	0

4. What is the square root of 276793836544 ?

OPERATION.

```
 5          27'67'93'83'65'44(526112
102         25
1046         267
10521        204
105221        6393
1052222       6276
               11783
               10521
                126265
                105221
                 2104444
                 2104444
                       0
```

5. What is the square root of 12321 ?

Ans. 111.

6. What is the square root of 53824 ?

Ans. 232.

7. What is the square root of 30858025 ?

Ans. 5555.

8. What is the square root of 16983563041 ?

Ans. 130321.

9. What is the square root of 852891037441 ?

Ans. 923521.

10. What is the square root of 61917364224 ?

Ans. 248832.

CASE II.

To extract the square root of a decimal fraction, or of a number consisting partly of a whole number, and partly of a decimal value, we have this

RULE.

I. Annex one cipher, if necessary, so that the number of decimals shall be even.

II. Then point off the decimals into periods of two figures each, counting from the units' place toward the right. If there are whole numbers, they must be pointed off as in Case I. Then extract the root, as in Case I.

NOTE.—If the given number has not an exact root, there will be a remainder after all the periods have been brought down, in which case the operation may be extended by forming new periods of ciphers.

EXAMPLES.

1. What is the square root of 3486.784401?
Ans. 59.049.
2. What is the square root of 25.62890625?
Ans. 5.0625.
3. What is the square root of 6.5536?
Ans. 2.56.
4. What is the square root of 0.00390625?
Ans. 0.0625.
5. What is the square root of 17?
Ans. 4.123 nearly.
6. What is the square root of 37.5?
Ans. 6.123 nearly.
7. What is the square root of 0.0000012321?
Ans. 0.00111.
8. What is the square root of 0.0011943936?
Ans. 0.03456.
9. What is the square root of 60.481729?
Ans. 7.777.

CASE III.

To extract the square root of a vulgar fraction, or mixed number, we have this

RULE.

I. Reduce the vulgar fraction, or mixed number, to its simplest fractional form.

II. Then extract the square root of the numerator and denominator separately, if they have exact roots; but when they have not, reduce the fraction to a decimal, and proceed as in Case II.

Repeat this rule.

EXAMPLES.

1. What is the square root of $\frac{256}{324}$?
Ans. $\frac{8}{9}$.

2. What is the square root of $\frac{10125}{12005}$?
Ans. $\frac{45}{49}$.

3. What is the square root of $4\frac{21}{25}$?
Ans. $2\frac{1}{5}$.

4. What is the square root of $\frac{5}{9}$ of $\frac{4}{5}$ of $\frac{4}{7}$ of $\frac{7}{9}$?
Ans. $\frac{4}{9}$.

5. What is the square root of $4\frac{1}{9}$?
Ans. 2.027 nearly.

6. What is the square root of $\frac{11}{17}$?
Ans. 0.8044 nearly.

7. What is the square root of $\frac{13}{49}$?
Ans. 0.052 nearly.

CASE IV.

When there are many decimal places required in the root, we may, after obtaining one more than half the number required, find the rest by dividing the remainder by the last TRUE DIVISOR, deprived of its right-hand figure.

EXAMPLES.

1. What is the square root of 11 to 10 decimals?

OPERATION.

```
3          11   (3.3166247903
63          9
661         2
6626      189
66326      11
66332|2     661
            439
            39756
             4144
             397956
              16444
              1326644
66332)317756(47903
       265328
        524280
        464324
         599560
         596988
          257200
          198996
           58204
```

EXPLANATION.

After obtaining five decimals in the root, by the usual method, we had 317756 for a remainder; the last true divisor was 663322, which, when deprived of its right-hand figure, becomes 66332. Hence, we divided the remainder 317756 by 66332, and found 47903 for a quotient; which, being written immediately after the figures already found, gives the root sought to 10 decimals.

2. What is the square root of 3 to 10 decimals?
Ans. 1.7320508075.

3. What is the square root of 0.00008876684 to 10 places of decimals? *Ans*. 0.0094216155.

4. What is the square root of 0.8867081113724 to 10 places of decimals? *Ans.* 0.9416517994.

EXAMPLES INVOLVING THE PRINCIPLES OF THE SQUARE ROOT.

119. *A triangle* is a figure having three sides, and consequently three angles.

When one of the angles is right, like the corner of a square, the triangle is called a *right-angled triangle.* In this case the side opposite the right angle is called the *hypotenuse.*

It is an established proposition of geometry, that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

From the above proposition, it follows that the square of the hypotenuse, diminished by the square of one of the sides, equals the square of the other side.

By means of these properties, it follows that two sides of a right-angled triangle being given, the third side can be found.

EXAMPLES.

1. How long must a ladder be, to reach to the top of a house 40 feet high, when the foot of it is 30 feet from the house?

In this example, it is obvious that the ladder forms the hypotenuse of a right-angled triangle, whose sides are 30 and 40 feet respectively. Therefore the square of the length of the ladder must equal the sum of the squares of 30 and 40.

$$30^2 = 900$$
$$40^2 = \underline{1600}$$
$$\sqrt{2500} = 50 \text{ the length of the ladder}$$

2. Suppose a ladder 100 feet long, to be placed 60 feet from the roots of a tree; how far up the tree will the top of the ladder reach? *Ans.* 80 feet.

3. Two persons start from the same place, and go, the one due north, 50 miles, the other due west, 80 miles. How far apart are they? *Ans.* 94.34 miles, nearly.

4. What is the distance through the opposite corners of a square yard? *Ans.* 4.24264 feet, nearly.

5. The distance between the lower ends of two equal rafters, in the different sides of a roof, is 32 feet, and the height of the ridge above the foot of the rafters is 12 feet. What is the length of a rafter? *Ans.* 20 feet.

6. What is the distance measured through the centre of a cube, from one corner to its opposite corner, the cube being 3 feet, or one yard, on a side?

Ans. 5.196 feet, nearly.

We know, from the principles of geometry, that all similar surfaces, or areas, are to each other as the squares of their like dimensions.

7. Suppose we have two circular pieces of land, the one 100 feet in diameter, the other 20 feet in diameter. How much more land is there in the larger than in the smaller?

By the above principle of geometry it follows, that the quantity of land in the two circles, must be as the squares of the diameters, that is, 100^2 to 20^2, or as 25 to 1. Hence, there is 25 times as much in the one piece, as there is in the other.

8. Suppose by observation, it is found that 4 gallons of water flow through a circular orifice of 1 inch in diameter in 1 minute. How many gallons would, under similar circumstances, be discharged through an orifice of 3 inches in diameter, in the same length of time?

Ans. 36 gallons.

9. What length of thread is required to wind spirally around a cylinder, 2 feet in circumference and 3 feet in length, so as to go but once around?

It is evident that if the cylinder be developed, or placed upon a plane, and caused to roll once over, that the convex surface of the cylinder will give a rectangle, whose width is 2 feet, and length 3 feet; at the same time the thread will form its diagonal. Hence, the length of the thread is $\sqrt{4+9}=\sqrt{13}=3.60555$ feet, nearly.

EXTRACTION OF THE CUBE ROOT.

120. If we cube 45 by the usual process, we find $45^3=91125$.

If, instead of 45, we take its equal $40+5$, and then cube it by actual multiplication, we shall have this

OPERATION.

$$\begin{array}{rrrr}
 & & 45=40+ & 5 \\
 & & 40+ & 5 \\
\hline
 & & 200+ & 25 \\
 & 1600+ & 200 & \\
\hline
 & 45^2=1600+ & 400+ & 25 \\
 & & 40+ & 5 \\
\hline
 & 8000+ & 2000+ & 125 \\
64000+ & 16000+ & 1000 & \\
\hline
45^3=64000+ & 24000+ & 3000+ & 125
\end{array}$$

Now, to reverse this process, that is, to extract the cube root of $64000+24000+3000+125$, we proceed as follows:

I. We first find the cube root of 64000 to be 40, which we place at the right of the number, in the form of a quotient in division, for the first part of the root sought.

We also place it on the left of the number, in a column headed 1st COL.; we next multiply it into itself, and place the result in a column headed 2d COL.; this last result, also multiplied by 40, gives 64000, which we subtract from the number, and obtain the remainder, $24000+3000+125$, which we will call the FIRST DIVIDEND.

II. We obtain the second term of the 1st column by adding the first term to itself, the result being multiplied by this first term, and added to the first term of the 2d column, gives its second term. Again, adding this first term to the second term of the 1st column, we get its third term.

III. We seek how many times the second term of the 2d column is contained in the first dividend, or simply how many times it is contained in its first part, 24000, which gives 5 for the second part of the root.

IV. Finally, we add this 5 to the last term of the 1st column, whose result, multiplied by 5, and added to the last term of the 2d column, gives its third term; which multiplied by 5, gives $27125=24000+3000+125$.

1ST COL.	2D COL.	Number.	Root
40	1600	$64000+24000+3000+125($	$40+5$
80	4800	64000	
120	5425	————————	
125		$24000+3000+125=27125$	
		27125	
		———	
		0	

This work can be written in a more condensed form, as follows, where the ciphers upon the right have been omitted.

1st col.	2d col.	Number. Root.
		91125 (45
4	16	64
8	48	27125
12	5425	27125
125		0

CASE I.

From the preceding operation, we may draw the following rule for extracting the cube root of a whole number.

RULE.

I. Since the cube of any number can not have more than three times as many places of figures as the number, we must separate the number into periods of three figures each, counting from the units' place towards the left. When the number of figures is not divisible by 3, the left-hand period will contain less than 3 figures.

II. Seek the greatest cube of the first, or left-hand period, place its root at the right of the number, after the manner of a quotient in division; also place it to the left of the number, for the first term of a column, marked 1st col. *Then multiply it into itself, and place the product for the first term of a column, marked* 2d col. *Again, multiply this last result by the same figure, and subtract the product from the first period, and to the remainder annex the next period, and it will give the* first dividend. *This same figure must be added to the first term of the 1st column; the sum will be its second term, which must be multiplied by the same figure, and the product added to the first term of the 2d column; the sum will be its second term, which we shall name the* first trial divisor.

The same figure of the root must be added to the second term of the 1st column, to form its third term.

III. See how many times the trial divisor, with two ciphers annexed, is contained in the dividend; the quotient

figure will be the second figure of the root, which must be placed at the right of the first figure; also annex it to the third term of the 1st column, and multiply the result by this second figure, and add the product, after advancing it two places to the right, to the last term of the 2d column. Again, multiply this last result by this second figure of the root, and subtract the product from the dividend, and to the remainder annex the next period, for a NEW DIVIDEND.

Proceed with this second figure of the root, precisely as was done with the first figure; and so continue until all the periods have been brought down.

NOTE.—I am aware that it is not an uncommon thing, to explain the principles on which the rules for extracting the square and cube roots are based, by geometry, instead of reversing the process of *involution*. Now, if we were desirous of forming a rule for the extraction of roots superior to the cube root, as, for instance, a rule for extracting the fifth root, we should be obliged to deduce it by reversing the operation of raising a number to the fifth power. In this case, geometry could be of no assistance; as a geometrical body can have only three dimensions. Since involution is purely an arithmetical operation, I see no good reason why its reverse operation of evolution should not also be purely arithmetical. No doubt the arithmetical rules for extracting the square and cube roots, may most easily be deduced from their corresponding algebraic rules. The rule here given for the cube root, is readily deduced from the algebraic rules as given in my algebra.

EXAMPLES.

1. Extract the cube root of 387420489.

OPERATION.

1st col.	2d col.		Number.	Root.
			387′420′489(729	
7	49		343	
14	147	1st trial divisor	44420	1st div.
212	15124		30248	
214	15552	2d trial divisor	14172489	2d div.
2169	1574721		14172489	
			0	

EXPLANATION.

The greatest cube of the first period, 387, is 343, whose root is 7, which we place to the right of the number for the first figure of the root sought. We also place it for the first term of the 1st column, which, multiplied into itself, gives $7\times7=49$, for the first term of the 2d column, which, in turn, multiplied by 7, gives $49\times7=343$, which, subtracted from the first period, 387, leaves the remainder, 44, to which, annexing the next period, 420, we get 44420, for the first dividend.

Again, adding 7 to the first term, 7, of 1st column, we get $7+7=14$, for the second term of the 1st column, which, multiplied by 7, gives $14\times7=98$; this, added to the first term of the 2d column, gives 147 for the second term of the 2d column, or the first trial divisor.

Again, adding 7 to the second term of the 1st column we get $14+7=21$, for the third term of the 1st column.

The trial divisor, with two ciphers annexed, becomes 14700, which is contained 3 times in the first dividend 44420. But the trial divisor being less than the true divisor, it will sometimes give too large a quotient figure; such is the case in this present example, where 2 is the second figure of the root.

The second figure 2 of the root, annexed to the third term of the 1st column, gives 212, which, multiplied by 2 gives 424, which, being advanced two places to the righ

must be added to 147, the last term of the second column. The sum 15124 will form the third term of the 2d column, which, multiplied by 2, gives $15124 \times 2 = 30248$, which, subtracted from the first dividend, leaves 14172, for the remainder, to which, annexing the next period, 489, we get 14172489, for the second dividend.

Again, to the last term, 212, of the 1st column, adding 2, we get 214 for the next term, which, multiplied by 2, gives 428, which added to 15124, gives 15552, for the second trial divisor. Again, adding 2 to 214, we get 216 for the fifth term of the 1st column.

The second trial divisor, with two ciphers annexed, becomes 1555200, which is contained 9 times in the second dividend, 14172489; therefore 9 is the third figure of the root, which, annexed to 216, gives 2169 for the last term of the 1st column, which, multiplied by 9, gives 19521, which, advanced two places to the right, and then added to 15552, gives 1574721; this, multiplied by 9, gives 14172489, which, subtracted from the second dividend, leaves no remainder.

2. What is the cube root of 913517247483640899?

OPERATION.

1ST COL.	2D COL.	Number.	Root.
		913′517′247′483′640′899(	970299
9	81	729	
18	243	184517	
277	26239	183673	
284	28227	844247483	
29102	282328204	564656408	
29104	282386412	279591075640	
291069	28241260821	254171347389	
291078	28243880523	25419728251899	
2910879	2824414250211	25419728251899	
		0	

3. What is the cube root of 10077696?
Ans. 216.

4. What is the cube root of 2357947691?
Ans. 1331.

5. What is the cube root of 42875?
Ans. 35.

6. What is the cube root of 117649?
Ans. 49.

7. What is the cube root of 7256313856?
Ans. 1936.

CASE II.

To extract the cube root of a decimal fraction, or of a number consisting partly of a whole number and partly of a decimal value, we have this

RULE.

I. Annex ciphers to the decimals, if necessary, so that the whole number may be divisible by 3.

II. Separate the decimals into periods of 3 *figures each, counting from the decimal point toward the right, and proceed as in whole numbers.*

NOTE.—If the given number has not an exact root, there will be a remainder after all the periods have been brought down. The process may be continued by annexing ciphers for new periods.

EXAMPLES.

1. What is the cube root of 0.469640998917?
Ans. 0.7773.

2. What is the cube root of 18.609625?
Ans. 2.65.

3. What is the cube root of 1.25992105?
Ans. 1.08005.

4. What is the cube root of 2?
Ans. 1.2599.

5. What is the cube root of 9?

Ans. 2.08008.

6. What is the cube root of 3?

Ans. 1.4422.

CASE III.

To extract the cube root of a vulgar fraction, or mixed number, we have this

RULE.

I. Reduce the fraction, or mixed number, to its simplest fractional form.

II. Extract the cube root of the numerator and denominator separately, if they have exact roots, but when they have not, reduce the fraction to a decimal, and then extract the root by Case II.

EXAMPLES.

1. What is the cube root of $\frac{2197}{4913}$?

Ans. $\frac{13}{17}$.

2. What is the cube root of $\frac{85169}{176723}$?

Ans. $\frac{22}{28}$.

3. What is the cube root of $17\frac{1}{8}$?

Ans. 2.577 nearly.

4. What is the cube root of $5\frac{1}{7}$?

Ans. 1.726 nearly.

5. What is the cube root of $\frac{81}{99}$?

Ans. 0.9353 nearly.

6. What is the cube root of $\frac{2}{3}$?

Ans. 0.8736 nearly.

EXAMPLES INVOLVING THE PRINCIPLES OF THE CUBE ROOT.

121. *It is an established theorem of geometry, that all similar solids are to each other as the cubes of their like dimensions.*

1. If a cannon-ball, 3 inches in diameter, weigh 8 pounds, what will a ball of the same metal weigh, whose diameter is 4 inches?

By the above theorem, we have

$$3^3 : 4^3 :: 8 \text{ pounds} : 18\tfrac{26}{27} \text{ pounds},$$

for the answer.

2. The celebrated Stockton gun, which, in bursting, proved so fatal to many of our distinguished citizens, is said to have carried a ball 12 inches in diameter, which weighed 238 pounds. What ought to be the diameter of another ball of the same metal, which should weigh 32 pounds?

$\frac{32}{238} \times 12^3 = 232.336$ inches nearly $=$ cube of the diameter of the ball sought.

Hence, $\sqrt[3]{232.336} = 6.1476$ inches nearly, the diameter of the ball required.

3. A cooper having a cask 40 inches long and 32 inches at the bung diameter, wishes to make another cask of the same shape, which shall contain just twice as much. What will be the dimensions of the new cask?

Ans. $\begin{cases} 40\sqrt[3]{2} = 50.3968 \text{ inches, nearly, for length.} \\ 32\sqrt[3]{2} = 40.3175 \text{ inches, nearly, for diameter.} \end{cases}$

4. What is the side of a cube, which will contain as much as a chest 8 feet 3 inches long, 3 feet wide, and 2 feet 7 inches deep? *Ans.* 47.984 inches, nearly.

5. How many cubic quarter inches can be made out of a cubic inch? *Ans.* 64.

6. Required the dimensions of a rectangular box, which

shall contain 20000 solid inches, the length, breadth, and depth, being to each other, as 4, 3, and 2.

Ans. { 37.641 inches, nearly.
28.231 " "
18.821 " " }

NOTE.—For a more complete treatise on the square and cube roots, as well as the roots of all powers, see Higher Arithmetic.

ARITHMETICAL PROGRESSION.

122. A series of numbers, which succeed each other regularly, by a common difference, is said to be in *arithmetical progression.*

When the terms are constantly increasing, the series is an *arithmetical progression ascending.*

When the terms are constantly decreasing, the series is an *arithmetical progression descending.*

Thus, 1, 3, 5, 7, 9, &c., is an ascending arithmetical progression; and 10, 8, 6, 4, 2, is a descending arithmetical progression.

In arithmetical progression, there are five things to be considered:

1. The first term.
2. The last term.
3. The common difference
4. The number of terms.
5. The sum of all the terms.

These quantities are so related to each other, that any three of them being given, the remaining two can be found.

Hence, there must be twenty distinct cases, arising from the different combinations of these five quantities.

To give a demonstration to all the rules of these twenty cases would be a very difficult task for the ordinary operations

of arithmetic: we will therefore content ourselves with demonstrating a few of the most important of them.

When are numbers in arithmetical progression? When is the progression ascending? When is it descending? Are the numbers 1, 3, 5, 7, 9, &c., in ascending or descending arithmetical progression? Mention the five quantities to be considered in arithmetical progression. How many of these must be given in order to be able to find the others? How many cases will thus arise?

CASE I.

By our definition of an ascending arithmetical progression, it follows, that the second term is equal to the first, increased by the common difference; the third is equal to the first, increased by twice the common difference; the fourth is equal to the first, increased by three times the common difference; and so on, for the succeeding terms.

Hence, when we have given the first term, the common difference, and the number of terms, to find the last term, we have this

RULE.

To the first term add the product of the common difference into the number of terms, less one.

EXAMPLES.

1. What is the 100th term of an arithmetical progression, whose first term is 2, and common difference 3?

In this example, the number of terms, less one, is 99, which, multiplied by the common difference, 3, gives 297, which, added to the first term, 2, makes 299 for the 100th term.

2. What is the 50th term of the arithmetical progression, whose first term is 1, the common difference $\frac{1}{2}$?

Ans. $25\frac{1}{2}$.

3. A man buys 10 sheep, giving $1 for the first, $3 for the second, $5 for the third, and so on, increasing in arithmetical progression. What did the last sheep cost at that rate?

Ans. $19.

4. The first term of an arithmetical progression is $\frac{3}{4}$ the common difference $\frac{1}{3}$, and the number of terms 25. What is the last term? *Ans.* $3\frac{1}{8}$

5. A person bought 100 yards of cloth; he gave 2*s.* 6*d* for the first yard, 2*s.* 10*d.* for the second yard, 3*s.* 2*d.* for the third yard; and so continuing to give 4*d.* more for each yard than he gave for the preceding one. How much did he give for the last yard? *Ans.* £1 15*s.* 6*d.*

CASE II.

From the nature of an arithmetical progression, we see that the second term added to the next to the last term is equal to the first added to the last; since the second term is as much greater than the first, as the next to the last is less than the last. After the same method of reasoning, we infer that the sum of any two terms equidistant from the extremes, is equal to the sum of the extremes.

Hence, it follows that the terms will average just half the sum of the extremes.

Therefore, when we have given the first term, the last term, and the number of terms, to find the sum of all the terms, we have this

RULE.

Multiply half the sum of the extremes by the number of terms.

EXAMPLES.

1. The first term of an arithmetical progression is 2, the last term is 50, and the number of terms is 17. What is the sum of all the terms?

In this example, half the sum of the extremes is

$$\frac{2+50}{2}=26.$$

This, multiplied by the number of terms, gives $26\times17=$ 442, for the sum required.

2. The first term of an arithmetical progression is 13, the last term is 1003, the number of terms is 100. What is the sum of the progression? *Ans.* 50800.

3. A person travels 25 days, going 11 miles the first day, 135 the last day; the miles which he travelled in the successive days, form an arithmetical progression. How far did he go in the 25 days? *Ans.* 1825 miles.

4. Bought 7 books, the prices of which are in arithmetical progression. The price of the first was 8 shillings, and the price of the last was 28 shillings. What did they all come to? *Ans.* £6 6*s*.

5. What is the sum of 1000 terms of an arithmetical progression, whose first term is 7 and last term 1113? *Ans.* 560000.

6. The first term of an arithmetical progression is $\frac{3}{7}$, and the last term $365\frac{4}{7}$, and the number of terms 799. What is the sum of all the terms? *Ans.* 146217.

7. What is the sum of 37 terms of an arithmetical progression, whose first term is 10, and last term 16? *Ans.* 481.

CASE III.

By Case I., we see that the last term is equal to the first term, increased by the product of the common difference into the number of terms, less one.

Hence, the first term must equal the last term, diminished by the product of the common difference into the number of terms, less one.

Therefore, when we have given the last term, the number of terms, and the common difference, to find the first term, we have this

RULE.

From the last term subtract the product of the common difference into the number of terms, less one.

EXAMPLES.

1. The last term of an arithmetical progression is 375, the common difference is 7, and the number of terms is 54. What is the first term?

In this example, the common difference, multiplied by the number of terms, less one, is $7 \times 53 = 371$, which, subtracted from the last term, gives $375 - 371 = 4$, for the first term.

2. The last term of an arithmetical progression is $39\frac{1}{3}$, the common difference is $\frac{2}{3}$, and the number of terms is 59. What is the first term? *Ans.* $\frac{2}{3}$.

3. The last term of an arithmetical progression is $49\frac{1}{2}$, the common difference is $1\frac{1}{2}$, and the number of terms 30. What is the first term? *Ans.* 6.

4. The last term of an arithmetical progression is 1008, the number of terms is 252, and the common difference is 4. What is the first term? *Ans.* 4.

5. A note is paid in 15 annual instalments; the payments are in arithmetical progression, whose common difference is 3; the last payment was \$49. What was the first payment? *Ans.* \$7.

CASE IV.

From Case I., we see that the product of the common difference into the number of terms, less one, is equal to the last term diminished by the first. Therefore, the difference of the last and first terms, divided by the common difference, is equal to the number of terms, less one.

Hence, when we have given the first term, the last term, and the common difference, to find the number of terms, we have this

RULE.

Divide the difference of the extremes by the common difference, and to the quotient, add one.

EXAMPLES.

1. The first term of an arithmetical progression is 5, the last term is 176, and the common difference 3. What is the number of terms?

In this example, the difference of the extremes is $176-5=171$; this, divided by the common difference, gives $\frac{171}{3}=57$; this, increased by 1, becomes 58, for the number of terms required.

2. The first term of an arithmetical progression is 11, the last term 88, and the common difference 7. What is the number of terms? *Ans.* 12.

3. A note becomes due in annual instalments, which are in arithmetical progression, whose common difference is 3; the first payment is \$7, the last payment is \$49. What is the number of instalments? *Ans.* 15.

CASE V.

We learn from Case I., that the product of the common difference into the number of terms, less one, is equal to the last term diminished by the first. Therefore, the difference of the last and first terms, divided by the number of terms, less one, will give the common difference.

Hence, when we have given the first term, the last term, and the number of terms, to find the common difference, we have this

RULE.

Divide the difference of the extremes by the number of terms, less one.

EXAMPLES.

1. The first term of an arithmetical progression is 5, the last term is 176, and the number of terms 58. What is the common difference?

In this example, the difference of the extremes is 171, which, divided by the number of terms, less one, becomes $\frac{171}{57}=3$, for the common difference.

2. The first term of an arithmetical progression is 17, the last term 3021, and the number of terms 752. What is the common difference? *Ans.* 4.

3. A person performs a journey in 17 days; the distances travelled on the successive days were in arithmetical progression; the first day he went 4 miles, and the last day he went 84. How many miles more did he go on each day, than on the preceding day?

Ans. 5 miles.

4. A man has seven sons, whose ages are in arithmetical progression; the age of the eldest is 41 years, the youngest is 5 years old. How many years is the common difference of their ages? *Ans.* 6.

CASE VI.

By Case II., we know that the sum of all the terms of an arithmetical progression is equal to half the sum of the extremes multiplied into the number of terms; therefore, the number of terms is equal to the sum of all the terms divided by half the sum of the extremes.

Hence, when we have given the first term, the last term, and the sum of all the terms, to find the number of terms, we have this

RULE.

Divide the sum of all the terms by half the sum of the extremes.

EXAMPLES.

1. The first term of an arithmetical progression is 1, the last term is 1001, and the sum of all the terms is 251001. What is the number of terms?

In this example, half the sum of the extremes is

$$\frac{1001+1}{2}=501.$$

Then, dividing the sum of all the terms by this, we obtain

$$\frac{251001}{501}=501,$$

for the number of terms.

2. In a triangular field of corn, the number of hills in the successive rows are in arithmetical progression: in the first row there is but one hill, in the last there are 81 hills; and the whole number of hills in the field is 1681. How many rows are there? *Ans.* 41 rows.

3. A man bought a certain number of yards of cloth for $152.50, giving 4 cents for the first yard, and increasing regularly on each succeeding yard, up to the last yard, for which he gave $3.01. How many yards of cloth did he purchase? *Ans.* 100 yards.

4. How many terms are there in an arithmetical progression whose first term is 5, last term 75, and sum of all the terms 440? *Ans.* 11.

CASE VII.

We also infer from Case II., that the sum of all the terms, divided by half the number of terms, will give the sum of the extremes. Therefore, if from the quotient of the sum of all the terms, divided by half the number of terms, we subtract the last term, we shall have left the first term.

Hence, when we have given the last term, the number of terms, and the sum of all the terms, to find the first term, we have this

RULE.

From the quotient of the sum of all the terms divided by half the number of terms, subtract the last term.

EXAMPLES.

1. If the last term of an arithmetical progression is 170, the number of terms 50, and the sum of all the terms 4450, what is the first term?

In this example, the sum of all the terms, divided by half the number of terms, is $\frac{445}{2\frac{1}{2}}=178$, from which subtracting the last term, we obtain $178-170=8$, for the first term.

2. A person wishes to discharge a debt of $1125 in 18 annual payments, which shall be in arithmetical progression. How much must his first payment be, so as to bring his last payment $120. *Ans.* $5.

3. What is the first term of an arithmetical progression whose number of terms is 27, last term 50, and sum of all the terms 729? *Ans.* 4.

4. The miles which a person travels in 19 successive days, form an arithmetical progression, whose last term is 80, the sum of all the terms 950. How many miles did he travel the first day? *Ans.* 20 miles.

CASE VIII.

From what has been said under Case VII., we infer that the first term, subtracted from the quotient of the sum of all the terms, divided by half the number of terms, will give the last term.

Hence, when we have given the sum of all the terms, the first term, and the number of terms, to find the last term, we have this

RULE.

From the quotient of the sum of all the terms, divided by half the number of terms, subtract the first term.

EXAMPLES.

1. If the first term of an arithmetical progression is 7, the number of terms 1000, and the sum of all the terms 560000, what is the last term?

In this example, the sum of all the terms, divided by half the number of terms, gives $\frac{560000}{500}=1120$, from

which subtracting the first term, we get $1120-7=1113$, for the last term.

2. If the first term of an arithmetical progression is 7, the number of terms 16, and sum of all the terms 142, what is the last term? *Ans.* $10\frac{3}{4}$.

3. The first term of an arithmetical progression is 13, the number of terms 100, and the sum of all the terms 50300. What is the last term? *Ans.* 993.

NOTE.—For the remaining cases of arithmetical progression, see Higher Arithmetic.

GEOMETRICAL PROGRESSION.

123. A series of numbers which succeed each other regularly, by a constant multiplier, is called a *geometrical progression.*

This constant factor, by which the successive terms are multiplied, is called the *ratio.*

When the ratio is greater than a unit, the series is called an *ascending geometrical progression.*

When the ratio is less than a unit, the series is called a *descending geometrical progression.*

Thus, 1, 3, 9, 27, 81, &c., is an ascending geometrical progression, whose ratio is 3.

And, 1 $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, &c., is a descending geometrical progression, whose ratio is $\frac{1}{4}$.

In geometrical progression, as in arithmetical progression, there are five things to be considered.

1. The first term.
2. The last term.
3. The common ratio.
4. The number of terms.
5. The sum of all the terms.

These quantities are so related to each other, that any three being given, the remaining two can be found.

Hence, there must be 20 distinct cases arising from the different combinations of these five quantities.

The solution of some of these cases require a knowledge of higher principles of mathematics than can be detailed by arithmetic alone.

We will give a demonstration of the rules of some of the most important cases.

When are numbers in geometrical progression? What is the constant factor, by which the successive terms are multiplied, called? When this ratio exceeds a unit, the progression is called what? When this ratio is less than a unit, how is the progression called? Give an example of an ascending geometrical progression. Give an example of a descending geometrical progression. How many quantities are to be considered in geometrical progression? Mention these quantities. How many of these must be known to enable us to find the others? How many cases will thus arise?

CASE I.

By the definition of a geometrical progression, it follows that the second term is equal to the first term, multiplied by the ratio; the third term is equal to the first term, multiplied by the second power of the ratio; the fourth term is equal to the first term, multiplied by the third power of the ratio; and so on, for the succeeding terms.

Hence, when we have given the first term, the ratio, and the number of terms, to find the last term, we have this

RULE.

Multiply the first term by the power of the ratio, whose exponent is one less than the number of terms.

EXAMPLES.

1. The first term of a geometrical progression is 1, the ratio is 2, and the number of terms is 7. What is the last term?

In this example, the power of the ratio, whose exponent is one less than the number of terms, is $2^6=64$, which, multiplied by the first term, 1, still remains 64, for the last term.

2. The first term of a geometrical progression is 5, the ratio is 4, and the number of terms 9. What is the last term? *Ans.* 327680.

3. A person travelling, goes 5 miles the first day, 10 miles the second day, 20 miles the third day, and so on, increasing in geometrical progression. If he continue to travel in this way for 7 days, how far will he go the last day? *Ans.* 320 miles.

CASE II.

If we multiply all the terms of a geometrical progression by the ratio, we shall obtain a *new* progression, whose first term equals the second term of the *old* progression; the second term of our *new* progression will equal the third term of the *old* progression, and so on for the succeeding terms.

Hence, the sum of the *old* progression, omitting the first term, equals the sum of the *new* progression, omitting its last term. The sum of the *new* progression is equal to the *old* progression, repeated as many times as there are units in the ratio.

Therefore, the difference between the *new* progression and the *old* progression, is equal to the *old* progression repeated as many times as there are units in the ratio, less one. But we also know, that the difference between these progressions is equal to the last term of the *new* progression, diminished by the first term of the *old* progression; and since the *new* progression was formed by multiplying the respective terms of the *old* progression by the

ratio, it follows that the last term of the *new* progression is equal to the last term of the *old* progression, repeated as many times as there are units in the ratio.

Therefore, the last term of the *new* progression, diminished by the first term of the *old* progression, is equal to the last term of the *old* progression, repeated as many times as there are units in the ratio, diminished by the first term of the *old* progression.

Hence, we finally obtain this condition:

That the sum of all the terms of a geometrical progression, repeated as many times as there are units in the ratio, less one, is equal to the last term, multiplied by the ratio, and diminished by the first term.

Therefore, when we have given the first term of a geometrical progression, the last term, and the ratio, to find the sum of all the terms, we have this

RULE.

Subtract the first term from the product of the last term into the ratio; divide the remainder by the ratio, less one.

EXAMPLES.

1. The first term of a geometrical progression is 4, the last term is 78732, and the ratio is 3. What is the sum of all the terms?

In this example, the first term subtracted from the product of the last term into the ratio, is 236192, which, divided by the ratio, less one, gives 118096, for the sum of all the terms.

2. The first term of a geometrical progression is 5, the last term is 327680, and the ratio is 4. What is the sum of all the terms?

Ans. 436905.

3. A person sowed a peck of wheat, and used the whole crop for seed the following year; the produce of this second year again for seed the third year, and so on. If, in

the last year, his crop is 1048576 pecks, how many pecks did he raise in all, allowing the increase to have been in a fourfold ratio?

Ans. 1398101 pecks.

CASE III.

Since by Case I., the last term is equal to the first term multiplied into a power of the ratio, whose exponent is equal to the number of terms, less one, it follows that the first term is equal to the last term, divided by the power of the ratio, whose exponent is one less than the number of terms.

Hence, when we have given the last term, the ratio, and the number of terms, to find the first term, we have this

RULE.

Divide the last term by a power of the ratio, whose exponent is one less than the number of terms.

EXAMPLES.

1. The last term of a geometrical progression is 1048576, the ratio is 4, and the number of terms is 11. What is the first term?

In this example, the ratio 4 raised to a power, whose index is 10, one less than the number of terms, is. $4^{10} = 1048576$; therefore 1048576, divided by 1048576, gives 1, for the first term.

2. A man has six sons, among whom he divides his estate in a geometrical progression, whose ratio is 2; the last son received $4800. How much did the first son receive?

Ans. $150.

3. A person bought 10 bushels of wheat, paying for it in a geometrical progression, whose ratio is 3; the last bushel cost him $196.83. What did he give for the first bushel?

Ans. 1 cent.

CASE IV.

We also discover from Case I., that the last term divided by the first term, will give the power of the ratio, whose exponent is the number of terms, less one.

Hence, when we have given the first term, the last term, and the number of terms, to find the ratio, we have this

RULE.

Divide the last term by the first term; extract that root of the quotient which is denoted by the number of terms, less one.

EXAMPLES.

1. The first term of a geometrical progression is 1, the last term is 64, and the number of terms is 7. What is the ratio?

In this example, the last term divided by the first term is 64, the number of terms, less one, is 6; therefore, we must extract the 6th root of 64; we first extract the square root, which is 8; we now extract the cube root of 8, which is 2, for the ratio.

2. In a country, during peace, the population increased every year in the same ratio, and so fast, that in the space of 5 years it became from 10000 to 14641 souls. By what ratio was the increase, yearly?

Ans. $\frac{11}{10}$.

3. The first term of a geometrical progression is 4, the last term is 78732, and the number of terms is 10. What is the ratio?

Ans. 3.

CASE V.

If, in Case II., we write the product of the first term, into the power of the ratio, whose exponent is the number of terms, less one, instead of the last term, as drawn from Case I., we shall have the sum of all the terms, repeated

as many times as there are units in the number of terms, less one, equal to the power of the ratio whose exponent is equal to the number of terms, diminished by one, and multiplied by the first term.

Hence, when we have given the first term, the ratio, and the number of terms, to find the sum of all the terms, we have this

RULE.

From the power of the ratio, whose exponent is the number of terms, subtract one; divide the remainder by the ratio, less one, and multiply the quotient by the first term.

EXAMPLES.

1. The first term of a geometrical progression is 3, the ratio is 4, and the number of terms 9. What is the sum of all the terms?

In this example, the ratio, raised to a power whose exponent is the number of terms, is $4^9=262144$; this, diminished by one, becomes 262143, which, divided by 3, gives 87381; this, multiplied by the first term, becomes $87381\times3=262143$, for the sum of all the terms.

2. A king in India, named Sheran, wished, according to the Arabic author Asephad, that Sessa, the inventor of chess, should himself choose a reward. He requested the grains of wheat which arise when 1 is calculated for the first square of the board, 2 for the second square, 4 for the third, and so on; reckoning for each of the 64 squares of the board, twice as many grains as for the preceding. When it was calculated, to the astonishment of the king, it was found to be an enormous number.

Ans. 18446744073709551615.

3. A gentleman married his daughter on New Year's day, and gave her husband 1 shilling towards her portion, and was to double it on the first day of every month during the year. What was her portion?

Ans. 4095*s.*

CASE VI.

We know from Case V., that the sum of all the terms multiplied by the ratio, less one, is equal to one subtracted from the power of the ratio, whose exponent is the number of terms, and this remainder multiplied by the first term.

Hence, when we have given the sum of all the terms, the number of terms, and the ratio, to find the first term, we have this

RULE.

Multiply the sum of all the terms by the ratio, less one, divide the product by the power of the ratio, whose index is the number of terms, after diminishing it by one.

EXAMPLES.

1. The sum of all the terms of a geometrical progression is 262143, the number of terms is 9, and the ratio is 4. What is the first term?

In this example, the sum of all the terms multiplied by the ratio, less one, is $262143 \times 3 = 786429$; the power of the ratio, whose exponent is the number of terms, is $4^9 = 262144$; this, diminished by 1, becomes 262143; therefore, 786429, divided by 262143, gives 3, for the first term.

2. The sum of all the terms of a geometrical progression is $591\frac{741}{4096}$, the number of terms is 7, and the ratio is $\frac{7}{4}$. What is the first term?

Ans. 9.

3. If a debt of $4095 is discharged in 12 months, by paying sums which are in geometrical progression, the ratio of which is 2, how much was the first payment?

Ans. $1.

CASE VII.

We have shown under Case II., that the sum of all the terms, multiplied by the ratio, less one, is equal to the first

term subtracted from the last term into the ratio; therefore, the first term is equal to the product of the ratio into the last term, diminished by the product of the ratio, less one, into the sum of all the terms.

Hence, when we have given the sum of all the terms, the last term, and the ratio, to find the first term, we have this

RULE.

Multiply the last term by the ratio, and from the product subtract the product of the sum of all the terms into the ratio, less one.

EXAMPLES.

1. The sum of all the terms of a geometrical progression is 436905, the last term is 327680, and the ratio is 4. What is the first term?

In this example, we find the first term, multiplied by the ratio, to be 1310720. The product of the sum of the terms into the ratio, less one, is 1310715; therefore, $1310720-1310715=5$, for the first term.

2. The sum of all the terms of a geometrical progression is 6138, the last term is 3072, and the ratio is 2. What is the first term?

Ans. 6.

3. The sum of all the terms of a geometrical progression is 1860040, the last term is 1240029, and the ratio is 3. What is the first term?

Ans. 7.

CASE VIII.

From the condition under Case II., we see that the ratio multiplied into the sum of all the terms diminished by the last term, is equal to the sum of all the terms, diminished by the first term.

Hence, when we have given the first term, the last term, and the sum of all the terms, to find the ratio, we have this

RULE.

Divide the sum of all the terms diminished by the first term, by the sum of all the terms diminished by the last term.

EXAMPLES.

1. The first term of a geometrical progression is 5, the last term is 327680, and the sum of all the terms is 436905. What is the ratio?

In this example, the sum of all the terms, diminished by the first term, is 436900, and the sum of all the terms, diminished by the last term, is 109225; therefore, 436900, divided by 109225, gives 4 for the ratio.

2. The first term of a geometrical progression is 6, the last term is 3072, and the sum of all the terms is 6138. What is the ratio?

Ans. 2.

3. The first term of a geometrical progression is 7, the last term is 1240029, and the sum of all the terms is 1860040. What is the ratio?

Ans. 3.

124. When the ratio of a geometrical progression is less than a unit, the first term will be the largest, and the last term the least; the progression will, in this case, be descending; but if we consider the series of terms in a reverse order, that is, calling the last term the first, and the first the last, the progression may then be considered as ascending.

If a decreasing geometrical progression be continued to an infinite number of terms, we may neglect the last term as of no appreciable value; we can find its sum by Case II., when it is modified, as follows:

Given the first term of a descending geometrical progression, and the ratio, to find the sum of all the terms, when continued to infinity.

RULE.

Divide the first term by a unit diminished by the ratio.

EXAMPLES.

1. What is the sum of all the terms of the infinite series $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, &c.?

In this example, a unit, diminished by the ratio, is $1-\frac{1}{2}=\frac{1}{2}$, and the first term, 1, divided by $\frac{1}{2}$, gives 2, for the sum of all the terms.

2. What is the sum of the infinite series $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$, &c.?

Ans. $1\frac{1}{2}$.

3. What is the sum of the infinite series $\frac{1}{10}, \frac{3}{100}, \frac{9}{1000}, \frac{27}{10000}$, &c.

Ans. $\frac{1}{7}$.

4. What is the sum of the infinite series $\frac{2}{10}, \frac{4}{100}, \frac{8}{1000}, \frac{16}{10000}$, &c.?

Ans. $\frac{1}{4}$.

5. What is the sum of the infinite series $\frac{1}{100}, \frac{3}{10000}, \frac{9}{1000000}$, &c.?

Ans. $\frac{1}{97}$.

6. What is the sum of the infinite series $\frac{1}{10}, \frac{6}{100}, \frac{36}{1000}, \frac{216}{10000}$, &c.?

Ans. $\frac{1}{4}$.

7. What is the sum of the infinite series $\frac{7}{100}, \frac{63}{10000}, \frac{567}{1000000}$, &c.?

Ans. $\frac{7}{91}$.

NOTE.—For further developments of geometrical progression, see Higher Arithmetic.

ALLIGATION.

125. Alligation is generally treated under two distinct heads, called *Alligation Medial* and *Alligation Alternate*. The latter, however, belongs properly to the province of Algebra.

ALLIGATION MEDIAL.

126. Alligation Medial teaches the method of finding the mean value of a compound, when its several ingredients and their respective values are given.

What is Alligation Medial?

Suppose a grocer mixes 140 pounds of tea, which is worth 8*s*. per pound; 200 pounds, worth 6*s*. per pound; and 160 pounds, worth 10*s*. per pound. What is a pound of the mixture worth?

140 pounds of tea, at 8*s*. per pound, is worth $140\times8=1120$*s*.; 200 pounds, at 6*s*., is worth $200\times6=1200$*s*.; 160 pounds, at 10*s*., is worth $160\times10=1600$*s*. Therefore, the mixture, which is 500 pounds, is worth $1120+1200+1600=3920$*s*. Hence, one pound of the mixture must be worth $\frac{3920}{500}=7\frac{21}{25}$*s*.

Hence, to find the mean value of a compound, composed of several ingredients of different values, we have this

RULE.

Divide the sum of the values of all the ingredients by the sum of the ingredients.

Repeat this Rule.

EXAMPLES.

1. A wine-merchant mixed several sorts of wine, viz.: 32 gallons at 40 cents per gallon; 15 gallons, at 60 cents

per gallon; 45 gallons, at 48 cents per gallon; and 8 gallons, at 85 cents per gallon. What is the value of a gallon of the mixture?

32	gallons, at	40	cents	=	$12.80
15	"	60	"	=	9.00
45	"	48	"	=	21.60
8	"	85	"	=	6.80
100	gallons of	mixture		=	$50.20

Therefore, one gallon of the mixture is worth $50.20÷100=$0.502=50 cents and 2 mills.

2. A farmer mixed together 7 bushels of rye, worth 72 cents per bushel; 15 bushels of corn, worth 60 cents per bushel; and 12 bushels of wheat, worth $1.20 per bushel. What is the value of a bushel of the mixture?

Ans. 0.83\frac{11}{17}$.

3. A goldsmith melts together 11 ounces of gold 23 carats fine, 8 ounces 21 carats fine, 10 ounces of pure gold, and 2 pounds of alloy. How many carats fine is the mixture? *Ans.* 12$\frac{25}{53}$.

It will be understood that a *carat* is a 24th part. Thus, 21 carats fine is the same as $\frac{21}{24}$ pure metal; in the same way, 23 carats fine is $\frac{23}{24}$ pure metal.

4. On a certain day, the mercury in the thermometer was observed to stand 2 hours at 62 degrees, 4 hours at 70 degrees, 5 hours at 72 degrees, 3 hours at 59 degrees, and 1 hour at 75 degrees. What was the mean temperature for the fifteen hours?

Ans. 67$\frac{11}{15}$ degrees.

5. Suppose a ship sail at the rate of 5 knots for 3 hours, at 7 knots for 5 hours, and 8 knots for 4 hours What is her rate of sailing during the 12 hours?

Ans. 6$\frac{5}{6}$ knots.

6. A grocer mixes 30 pounds of sugar worth 10 cents per pound; 40 pounds worth 10$\frac{1}{2}$ cents per pound; 24 pounds worth 11 cents per pound; and 60 pounds worth 13 cents per pound. What is a pound of the mixture worth?

Ans. 11$\frac{5}{11}$ cents.

ALLIGATION ALTERNATE.

127. ALLIGATION ALTERNATE is the reverse of Alligation Medial; that is, it teaches the method of finding the ingredients when their rates are given, so that the compound shall have a given value.

What is Alligation Alternate?

Suppose we wish to mix teas, which are worth 4 and 6 shillings per pound, so that the mixture may be worth 5 shillings per pound; it is obvious that we must take equal quantities of each; since the price of the one is as much less than the mean price, as the other is greater.

Again, suppose we wish to mix teas, which are worth 4 and 7 shillings per pound, so that the mixture may be worth 5 shillings. In this case the 7 shilling tea is 2 shillings above the average price, whilst the 4 shilling tea is but 1 shilling below: it will be necessary to use twice as much of the 4 shilling tea as of the 7 shilling tea; and in all cases it is obvious that the quantities to be used will be in the inverse ratio to the differences between their prices and the mean price.

When there are more than two simples they may be compared together in couplets, one term of which must exceed the average price, while the other must be less.

CASE I.

The rates of the several ingredients being given, to make a compound of a fixed rate.

From what has been said above, we draw the following

RULE.

I. Write the rates of the simples in a line under each other, then connect each rate of the ingredients which is less than the rate of the compound, with one or more rates

greater than the rate of the compound; connect in the same way, each rate which is greater than the rate of the compound, with one or more rates which are less.

II. Write the difference between each rate of the ingredients and the compound rate, opposite the rate of the ingredients with which it is connected. If only one difference stands against any rate, it will be the required quantity of the ingredient of that rate; but if there be several, their sum will be the quantity required.

Repeat this Rule.

EXAMPLES.

1. How much sugar at 5, 6, and 10 cents per pound, must be mixed together, so that a pound of the mixture may be worth 8 cents?

SOLUTION.

$$8\left\{\begin{array}{l} 5 \\ 6 \\ 10 \end{array}\right. \qquad \begin{array}{r} 2 \\ 2 \\ 3+2=5 \end{array}$$

Therefore, if we take 2 pounds at 5 cents, 2 pounds at 6 cents, and 5 pounds at 10 cents, we shall satisfy the conditions of the question. It is obvious, that any other number of pounds which are to each other as the numbers 2, 2, and 5, will satisfy the question equally well; so that in Alligation Alternate the number of solutions are *indefinite;* all that we can do is to find the ratios of the quantities required.

NOTE.—In many cases the ingredients will admit of being connected in several ways, and then we shall obtain as many sets of ratios as there are methods of connecting them.

2. How many pounds of raisins at 4, 6, 8, and 10 cents per pound, must be mixed, so that a pound of the compound may be worth 7 cents?

In this question, the terms may be connected in seven distinct ways; therefore, we shall obtain seven sets of ratios, as follows:

$$7\left\{\begin{array}{c}4\\6\\8\\10\end{array}\right.\begin{array}{r}3\\1\\1\\3\end{array}\qquad 7\left\{\begin{array}{c}4\\6\\8\\10\end{array}\right.\begin{array}{r}1\\3\\3\\1\end{array}\qquad 7\left\{\begin{array}{c}4\\6\\8\\10\end{array}\right.\begin{array}{r}1+3=4\\1\\3+1=4\\3\end{array}$$

$$7\left\{\begin{array}{c}4\\6\\8\\10\end{array}\right.\begin{array}{r}3\\1+3=4\\1\\3+1=4\end{array}\qquad 7\left\{\begin{array}{c}4\\6\\8\\10\end{array}\right.\begin{array}{r}1\\1+3=4\\3+1=4\\1\end{array}$$

$$7\left\{\begin{array}{c}4\\6\\8\\10\end{array}\right.\begin{array}{r}1+3=4\\3\\3\\3+1=4\end{array}\qquad 7\left\{\begin{array}{c}4\\6\\8\\10\end{array}\right.\begin{array}{r}1+3=4\\1+3=4\\3+1=4\\3+1=4\end{array}$$

3. How much wine, at 72 cents per gallon, and 48 cents per gallon, must be mixed together, that the composition may be worth 60 cents per gallon?

Ans. An equal quantity of each.

4. How many gallons of wine and water must be mixed together, so that the mixture may be worth 60 cents per gallon, the water being considered of no value, and the wine with which it is mixed being worth 90 cents per gallon?

Ans. 2 gallons of wine to 1 of water.

5. Having gold of 12, 16, 17, and 22 carats fine, what proportion of each kind must I take, to make a compound of 18 carats fine?

Ans. 4, 4, 4, 9.

6. It is required to mix different sorts of grain, at 56, 62, and 75 cents per bushel, so that the mixture may be worth 60 cents per bushel. How much of each kind must be taken?

Ans. 17, 4, 4.

CASE II.

When one of the ingredients is limited to a certain quantity, we have this

RULE.

Find the proportionate quantities of each ingredient, by Case I., in the same manner as though there was no limitation; then as the difference against the simple whose quantity is given, is to each of the other differences, so is the given quantity of that simple to the quantity required of each of the other simples.

Repeat this Rule.

EXAMPLES.

1. A person wishes to mix 10 bushels of wheat worth \$1 per bushel, with rye, worth 70 cents per bushel, and oats worth 30 cents per bushel, so that the mixture may be worth 60 cents per bushel. How many bushels of rye and oats must he use?

Proceeding according to Case I., we find the proportionate numbers to be 30, 30, and 50. Hence,

$$30 : 30 :: 10 : 10$$
$$30 : 50 :: 10 : 16\tfrac{2}{3}$$

So that he must make use of 10 bushels of rye, and $16\frac{2}{3}$ bushels of oats.

2. A grocer has 90 pounds of tea, worth 90 cents per pound, which he wishes to mix with three other qualities, valued at 80 cents, 70 cents, and 60 cents, per pound. How much must he take of these three kinds so as to be able to sell the mixture at 85 cents per pound?

Ans. 10 pounds of each.

3. A merchant has 90 pounds of spice worth 86 cents per pound, which he wishes to mix with three other sorts which are worth 30, 40, and 50 cents per pound, respectively. How many pounds must be used so that the compound may be worth 55 cents per pound?

Ans. He must use 62 pounds of each.

CASE III

When two or more of the ingredients are limited in quantity, we have the following

RULE.

Find, as in Alligation Medial, what will be the rate of a mixture made of the given quantities of the limited ingredients only; then consider this as the rate of a limited ingredient, whose quantity is the sum of the quantities of the limited ingredients, from which, and the rates of the unlimited ingredients, proceed to calculate the several quantities required, as in Case II.

Repeat this Rule.

EXAMPLES.

1. I have 36 gallons of wine at 24 cents a gallon, 8 gallons at 52 cents, and 4 gallons at 88 cents, and would mix the whole with three other kinds of wine, one at $1.25, one at 86 cents, and the other at 90 cents per gallon. How many gallons of the last sort must I use, so that the mixture may be worth $1 per gallon?

By Alligation Medial, we find as follows:

36	gallons at	24	cents	=	$8.64
8	"	52	"	=	4.16
4	"	88	"	=	3.52
48	gallons come to				$16.32

Therefore, one gallon of this mixture is worth 34 cents.

Now by Case II., of Alligation Alternate, we have

$$100\left\{\begin{array}{l} 34 \\ 125 \\ 86 \\ 90 \end{array}\right. \quad \begin{array}{r} 25 \\ 66+14+10=90 \\ 25 \\ 25 \end{array}$$

$$25 : 90 :: 48 : 172\tfrac{4}{5}$$
$$25 : 25 :: 48 : 48$$
$$25 : 25 :: 48 : 48$$

Therefore, I must take 48 gallons of the two sorts, which are worth 86 and 90 cents per gallon, and $172\frac{4}{5}$ gallons of the sort which is worth $1.25 per gallon.

2. With 63 gallons of wine worth 8 shillings per gallon, and 126 gallons worth 11 shillings per gallon, I mixed other wine at 7 shillings, and some water: I then found that it was worth 6 shillings per gallon. How much wine and water did I mix with the 189 gallons?

Ans. 189 gallons of wine, and $157\frac{1}{2}$ of water.

3. A person wishes to mix 10 bushels of wheat at 70 cents per bushel, and 12 bushels of rye at 48 cents per bushel, with corn at 36 cents, and barley at 30 cents per bushel, so that a bushel of the mixture may be worth 38 cents. What quantity of each must be taken?

Ans. 44 bushels of each.

4. With 50 pounds of tea worth 75 cents per pound, and 40 pounds worth $79\frac{1}{2}$ cents per pound, I wish to mix other teas worth 90 and 95 cents per pound, so as to be able to afford the mixture at 80 cents per pound. How many pounds of the 90 and 95 cent teas must I use?

Ans. $10\frac{4}{5}$ pounds of each.

CASE IV.

When the whole compound is limited to a certain quantity, we have this

RULE.

Find the proportional parts, as in Case I.; then as the sum of the proportional parts thus obtained, is to the given quantity, so is the proportionate quantity of each ingredient to its required quantity.

Repeat this Rule.

EXAMPLES.

1. Having three sorts of raisins at 9, 12, and 18 cents per pound, what quantity of each sort must I take to fill a cask of 210 pounds, so that its contents may be worth 14 cents per pound?

SOLUTION.

$$14\left\{\begin{array}{l}9\\12\\18\end{array}\right. \qquad \begin{array}{r}4\\4\\5+2=7\end{array}$$

$$\begin{array}{rcrcrcr}
15 & : & 210 & :: & 4 & : & 56\\
15 & : & 210 & :: & 4 & : & 56\\
15 & : & 210 & :: & 7 & : & \underline{98}\\
 & & & & & & \underline{210}
\end{array}$$

Therefore, I must take 56 pounds each, at 9 and 12 cents per pound, and 98 pounds at 18 cents per pound.

2. A goldsmith has gold of 14, 18, and 20 carats fine, and would mix of all these sorts so much as to make a mass of 50 ounces, which shall be 16 carats fine. How much of each sort is required?

Ans. { 30 ounces at 14 carats fine; and 10 ounces each, at 18 and 20 carats fine.

3. How much water must be mixed with brandy, worth \$1.60 per gallon, to reduce the price to \$1.20 per gallon provided it fill a cask of 120 gallons?

Ans. 30 gallons

DUODECIMALS.

128. In decimals we have seen that the figures decrease in a tenfold ratio, from the left towards the right.

In duodecimals, this decrement goes on in a twelvefold ratio.

The different denominations are the *foot* (*f.*), the *prime*, or inch ($'$), the *second* ($''$), the *third* ($'''$), the *fourth* ($''''$), the *fifth* ($'''''$), and so on.

Thus, 7*f.*, 6$'$, 3$''$, 4$'''$, 5$''''$, is read 7 feet, 6 primes, 3 seconds, 4 thirds, 5 fourths.

The accents used to distinguish the denominations below feet, are called *indices.*

Taking the foot for the unit, we have the following relations:

$1' = \frac{1}{12}$ of 1 foot.
$1'' = \frac{1}{12}$ of $\frac{1}{12}$ of 1 foot $= \frac{1}{144}$ of 1 foot.
$1''' = \frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of 1 foot $= \frac{1}{1728}$ of 1 foot.
$1'''' = \frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of 1 foot $= \frac{1}{20736}$ of 1 foot.
&c. &c. &c. &c.

ADDITION AND SUBTRACTION OF DUODECIMALS.

129. Addition and Subtraction of duodecimals, are performed like addition and subtraction of other denominate numbers, remembering that 12 of any denomination make one of the next greater denomination.

In decimals how do figures decrease from the left toward the right? In duodecimals how do they decrease? What are the different denominations of duodecimals? What are the accents called which are used to distinguish the different denominations below the foot? How is addition and subtraction of duodecimals performed?

EXAMPLES.

(1.)				(2.)				
17f.	7′	8″		365f.	1′	7″	9‴	
25f.	0′	2″		521f.	10′	10″	11‴	
30f.	10′	11″		605f.	8′	8″	1‴	
29f.	6′	6″		731f.	3′	0″	8‴	
103f.	1′	3″	Sum.	2224f.	0′	3″	5‴	Sum.

3. What is the sum of 3*f.* 6′ 4″, 8*f.* 3′ 4″, 9*f.* 1′ 3″, and 10*f.* 10′ 10″?

Ans. 31*f.* 9′ 9″.

4. What is the sum of 100*f.* 8′ 8″, 135*f.* 0′ 1″, 65*f.* 9′ 2″, 45*f.* 3′ 3″, and 200*f.* 6′ 6″?

Ans. 547*f.* 3′ 8″.

	(5.)			(6.)		
From	87f.	3′	4″	100f.	10′	10″
Subtract	35f.	8′	9″	90f.	6′	3″
Remainder	51f.	6′	7″	10f.	4′	7″

7. From 25*f.* 6′ 6″ subtract 18*f.* 9′ 10″.

Ans. 6*f.* 8′ 8″

8. From 100*f.* subtract 58*f.* 2′ 1″.

Ans. 41*f.* 9′ 11″.

MULTIPLICATION OF DUODECIMALS.

130. Suppose we wish to multiply 14*f.* 7′ by 2*f.* 3′, we should proceed as follows:

	14f.	7′	
	2f.	3′	
	3f.	7′	9″
	29f.	2′	
Ans.	32f.	9′	9″ $=32f.+\frac{9}{12}$ of a foot $+\frac{9}{144}$ of a foot

EXPLANATION.

We begin on the right hand, and multiply the multiplicand through, first by the primes of the multiplier, then by the feet of the multiplier, thus: $3' \times 7' = \frac{3}{12} \times \frac{7}{12} = \frac{21}{144}$ of a foot, which is $21'' = 1'\ 9''$; we write down the $9''$, and carry the $1'$ to the next product; again, $14f. \times 3' = 14 \times \frac{3}{12} = \frac{42}{12}$ of a foot, which is $42'$; now adding in the $1'$, which was to carry from the last product, we have $43' = 3f.\ 7'$, which we write down, thus finishing the first line of products.

Again, we have $2f. \times 7' = 2 \times \frac{7}{12} = \frac{14}{12}$ of a foot, which is $14' = 1f.\ 2'$; we write the $2'$ under the seconds of the last line, and carry $1f.$ to the next product; $2f. \times 14f. = 28f.$, to which, adding in the $1f.$, which was to carry from the last product, we have $29f.$, which we place underneath the feet of the last line. Taking the sum, we find $32f.\ 9'\ 9''$, for the answer.

From the above we infer, *that if we consider the index of the feet to be 0, then the denomination of each product will be denoted by the sum of the indices, representing the factors.*

Thus, *feet* by *feet*, produces *feet*; *feet* by *primes*, produces *primes*; *primes* by *primes*, produces *seconds*, &c.

Hence, to multiply a number consisting of feet, inches, seconds, &c., by another number consisting of like qualities, we have this

RULE.

Place the several terms of the multiplier under the corresponding ones of the multiplicand. Beginning at the right hand, multiply the several terms of the multiplicand by the several terms of the multiplier successively, placing the right hand term of each of the partial products under its multiplier; then add the partial products together, observing to carry one for every twelve, both in multiplying and adding. The sum of the partial products will be the answer

Repeat this Rule.

EXAMPLES.

1. What is the product of $3f.\ 7'\ 2''$ by $7f.\ 6'\ 3''$?

OPERATION.

			3f.	7'	2''
			7f.	6'	3''
			10''	9'''	6''''
	1f.	9'	7''	0'''	
	25f.	2'	2''		
Ans.	27f.	0'	7''	9'''	6''''

2. What is the product of $7f.\ 6'\ 4''$ by $2f.\ 3'\ 5''$?

OPERATION.

			7f.	6'	4''
			2f.	3'	5''
		3'	1''	7'''	8''''
	1f.	10'	7''	0'''	
	15f.	0'	8''		
Ans.	17f.	2'	4''	7'''	8''''

3. What is the product of $7f.\ 8'$ by $6f.\ 4'\ 3''$?

Ans. $48f.\ 8'\ 7''$.

4. What is the product of $6f.\ 9'\ 7''$ by $4f.\ 2'$?

Ans. $28f.\ 3'\ 11''\ 2'''$.

5. What is the area of a marble slab, whose length is $7f.\ 3'$, and breadth $2f.\ 11'$?

Ans. $21f.\ 1'\ 9''$.

6. How many square feet are contained in the floor of a hall $37f.\ 3'$ long, by $10f.\ 7'$ wide?

Ans. $394f.\ 2'\ 9''$.

7. How many square feet are contained in a garden $100f.\ 6'$ in length, by $39f.\ 7'$ in width?

Ans. $3978f.\ 1'\ 6''$.

8. How many yards of carpeting, one yard in width, will it require to cover a room $16f.\ 5'$ by $13f.\ 7'$?

Ans. $222f.\ 11'\ 11''$.

9. What will the plastering of a ceiling cost at 13 cents a square yard, its length being 30*f.* 7 inches, and the breadth 22*f.* 4 inches?

Ans. \$9.86, nearly.

10. How many cubic feet are contained in a rectangular stone, 7*f.* 4′ long, 2*f.* 11′ wide, and 1*f.* 10′ thick?

Ans 39*f.* 2′ 6″ 8‴.

PROMISCUOUS QUESTIONS.

1. Suppose I purchase \$1200 worth of goods, $\frac{1}{3}$ of which is on a credit of 3 months, $\frac{1}{3}$ on a credit of 6 months, and the remaining $\frac{1}{3}$ on a credit of 9 months. How much ready money ought to pay the purchase, interest being 7 per cent.?

Ans. \$1159.64, nearly.

2. In the above example, by the principles of equation of payments, how much credit ought I to have on the whole sum of \$1200?

Ans. 6 months.

3. Now, what is the present worth of \$1200 due at the end of 6 months, interest being 7 per cent.?

Ans. \$1159.42, nearly.

4. I employed A and B to ditch my meadow. A was to receive $87\frac{1}{2}$ cents per rod, and B was to have $112\frac{1}{2}$ cents per rod; each wrought until his wages amounted to \$50. What was the amount of ditch dug by both?

Ans. $101\frac{37}{63}$ rods.

5. Three merchants, A, B, and C, enter into partnership. A advances \$1200, B \$800, and C \$600. A leaves his money 8 months, B 10 months, and C 14 months in the business. They gain \$500. What is the share of each?

$$Ans. \begin{cases} \text{A receives } \$184\frac{8}{13}. \\ \text{B} \quad " \quad 153\frac{11}{13}. \\ \text{C} \quad " \quad 161\frac{7}{13}. \end{cases}$$

6. A and B have the same income; A saves $\frac{1}{5}$ of his; but B, by spending $120 per annum more than A, at the end of 10 years finds himself $200 in debt. What was the income?

Since A saves $\frac{1}{5}$, he must spend $\frac{4}{5}$.

Now, as B's debt amounts to $200 in ten years, it must be $20 in one year; hence, if B had spent $20 per year less than he did, he would neither have run into debt, nor have saved anything. Hence, $120—$20=$100, was $\frac{1}{5}$ of his income; the income was therefore $500.

7. Suppose a book to contain 365 pages, averaging 40 lines of 10 words each on each page. How many words would the book contain?

Ans. 146000 words.

8. There are 31173 verses in the Bible; how many days will it require to read it through, if 30 verses are read daily?

Ans. $1039\frac{1}{10}$ days.

9. After expending $\frac{1}{4}$ of my money, and $\frac{1}{4}$ of the remainder, I had remaining $72; how much had I at first?

Ans. $128.

10. If I sell cloth at $1.50 per yard, and gain 25 per cent., how ought I to have sold it so as to lose 20 per cent.?

Ans. $0.96.

11. Sold cloth at $1.50 per yard, and gained 25 per cent. What should I have lost per cent., if I had sold it at $0.96 per yard?

Ans. 20 per cent.

12. If I buy cloth at $1.20 per yard, how must I sell it so as to gain 25 per cent.?

Ans. $1.50.

13. A merchant has to make the following payments at three different periods: $2832 in 3 months, $2560 in 9 months, and $1450 in 16 months. The creditor wishes to receive the whole sum of $6842 at once. When ought the payment to be made?

Ans. In 8 months.

14. A father gives to his five sons \$1000, which they are to divide according to their ages, so that each elder son shall receive \$20 more than his next younger brother. What is the share of the youngest?

Ans. \$160.

15. A company of 90 persons, consists of men, women, and children. The men are 4 in number more than the women, the children 10 more than the adults. How many men, women, and children, are there in the company?

Ans. { 22 men, 18 women, 50 children

16. The common school fund for the state of New-York was \$1975093.15 in 1843, and during the same year there were in the state, 677995 children between the ages of 5 and 16 years. How much would the above fund amount to per scholar?

Ans. \$2.91, nearly.

17. The whole number of volumes in the common school libraries in 1843, was 874865. What would be their value, if they are estimated at $37\frac{1}{2}$ cents per volume?

Ans. \$328074.375.

18. The whole number of children taught during the year 1843, was 657782, and the whole number of schools was 10860. How many scholars on an average would each school consist of?

Ans. Between 60 and 61.

19. Suppose the Erie canal to be 60 feet wide, and 6 feet deep; how many miles in length will it require to make one cubic mile of water?

A mile being 5280 feet, it follows that a mile in length of the canal contains $5280 \times 60 \times 6$ cubic feet of water.

A cubic mile consists of $5280 \times 5280 \times 5280$ cubic feet.

Hence, the number of miles in length of canal required is

$$\frac{5280 \times 5280 \times 5280}{5280 \times 60 \times 6} = 880 \times 88 = 77440 \text{ miles,}$$

which is more than 3 times the distance around the world.

20. A person owning $\frac{3}{5}$ of a copper mine, sells $\frac{3}{4}$ of his interest in it for \$1800. What, at this rate, was the value of the whole?

Ans. \$4000.

21. Suppose I buy a certain lot of oranges at 3 cents a piece, and as many more at 5 cents a piece, and sell them at 4 cents a piece; do I gain or lose by the operation?

Ans. I neither gain nor lose

22. Suppose I buy a certain number of oranges at 3 for one cent, and as many more at 5 for one cent, and sell them at 4 for one cent; do I gain or lose by the operation?

Ans. I lose $\frac{1}{60}$ of a cent on each orange. If the whole number of oranges was 60, I should lose one cent.

23. Suppose I expend a certain sum of money for oranges at 3 cents a piece, and another equal sum, for another lot at 5 cents a piece; how much do I gain on each cent expended, if I sell them all at 4 cents a piece?

Ans. I gain $\frac{1}{15}$ of a cent on each cent employed in the purchase. If the whole sum employed was 15 cents, I should gain 1 cent.

24. If A can can do a piece of work in 3 days, B in 4 days, and C in 5 days, how much longer will it take B to do it alone, than it will take A and C together to do it?

Ans. $2\frac{2}{15}$ times.

25. If A can accomplish a piece of work in $\frac{1}{3}$ of a day, B in $\frac{1}{4}$ of a day, and C in $\frac{1}{5}$ of a day, how much longer will it take B to do it alone, than it will take A and C together to do it?

Ans. 2 times.

26. What is the shortest piece of cloth which shall be at the same time, an even number of yards, an even number of Ells Flemish, an even number of Ells English, and an even number of Ells French?

Seek the least multiple of 4, 3, 5, 6.

Ans. 60 quarters$=$15 yards

27. A man died, leaving $1000, to be divided between his two sons, one 14, and the other 18 years of age, in such a proportion, that the share of each being put to interest at 6 per cent., should amount to the same sum when they should arrive at the age of 21. What did each one receive?

Since the shares of each would amount to equal sums when they should come of age, it is obvious that they must have been to each other reciprocally as the amount of $1 for the respective times 7 years and 3 years. The amount of $1 for 7 years at 6 per cent., is $1.42; the amount of $1 for 3 years at 6 per cent., is $1.18. Hence, their portions were as 118 is to 142, or as 59 to 71. The sum of these numbers is 130. Therefore,

The younger must have $\frac{59}{130}$ of $1000=$453.846, nearly.
The elder must have $\frac{71}{130}$ of $1000=$546.154, nearly.

28. Divide $100 between A, B, and C, so that B may have $3 more than A, and C $4 more than B. How much must each one have?

Ans. A has $30. B " 33. C " 37.

29. A can do a piece of work in 4 days, and B can do the same in 3 days. How long would it take both together to do it?

Ans. $1\frac{5}{7}$ days.

30. A person wishes to dispose of his horse by lottery. If he sells the tickets at $2 each, he will lose $30 on his horse; but if he sells them at $3 each, he will receive $30 more than his horse cost him. What is the value of the horse, and the number of tickets?

Since the number of tickets at $2 each, is $30 less than the value of the horse, and at $3 each is $30 more than the value of the horse, it follows that the tickets will amount to $60 more when estimated at $3 than when estimated at $2. Now the difference on one ticket is $1; consequently, there must have been 60 tickets.

60 tickets at $2 each, will amount to $120, to which adding $30, we get $150 for the value of the horse.

31. Thomas sold 150 pine-apples at $33\frac{1}{3}$ cents a piece, and received the same amount of money that Henry did for water-melons at 25 cents a piece. How much money did each receive, and how many melons did Henry sell?

Ans. Each received \$50, and Henry sold 200 melons.

32. A man bought apples at 5 cents a dozen, half of which he exchanged for pears, at the rate of 8 apples for 5 pears; he then sold all his apples and pears at a cent a piece, and thus gained 19 cents. How many apples did he buy, and how much did they cost?

Had he bought 16 apples, they would have cost him $\frac{5}{12}$ of $16=\frac{20}{3}=6\frac{2}{3}$ cents.

Then he must exchange 8 apples for 5 pears, so that his apples and pears together will be 13, which, at a cent a piece, will amount to 13 cents; the whole cost was $6\frac{2}{3}$ cents. Therefore, by purchasing 16 apples he makes $6\frac{1}{3}$ cents, but he made 19 cents, which is 3 times $6\frac{1}{3}$; therefore, he must have bought 3 times 16 apples, that is, he purchased 48 apples, whose cost must have been 20 cents.

33. A person expended \$23.40 for eggs. With one half of his money he purchased a lot at 13 cents per dozen; with the other half of his money he purchased another lot at 9 cents per dozen. He afterward sold them all together at 11 cents per dozen. Did he gain or lose by the operation?

Ans. He gained 80 cents.

34. Divide \$1200 between A and B so that A's share may be to B's as 2 to 7.

Ans. { A has $\$266\frac{2}{3}$.
B has $\$933\frac{1}{3}$. }

35. A gentleman spends $\frac{2}{3}$ of his yearly income for board and lodging, $\frac{2}{3}$ of the remainder for clothes, and $\frac{2}{3}$ of what now remains he bestows for charitable purposes, and saves \$100 yearly. What was his income?

Ans. \$2700.

36. If I buy an article for \$4, and sell it for \$5, how much per cent. do I gain?

Ans. 25 per cent.

37. If I give \$5 for an article, and sell it for \$4, how much per cent. do I lose?

Ans. 20 per cent.

38. What is the interest of \$175 for 3 months, at 6 per cent.?

Ans. \$2.625.

39. How many yards of Brussels carpeting, which is $\frac{3}{4}$ of a yard wide, will it require to cover a floor 18 feet by 20 feet?

Ans. $53\frac{1}{3}$ yards.

40. Admitting the velocity of a cannon-ball to be 1600 feet per second, what time, at this velocity, would it require to move 95 millions of miles, which is the distance from the earth to the sun, counting $365\frac{1}{4}$ days to the year?

Ans. $9\frac{12284}{13149}$ years.

41. The Winchester bushel measure is of a cylindric form, 8 inches deep, and $18\frac{1}{2}$ inches in diameter, containing $2150\frac{2}{5}$ cubic inches. What must be the side of a cubical box which shall contain the same quantity?

The cube root of $2150\frac{2}{5}=12.907$, nearly, for the length of a side, in inches.

42. The clocks of Italy go on to 24 hours; then how many strokes do they strike in one revolution of the index?

Ans. 300.

43. There is an island 20 miles in circumference, and three men, A, B, and C, start from the same point, and travel the same way about it; A goes 3 miles per hour, B goes 7 miles per hour, and C goes 11 miles per hour. In what time will they all be together?

Since B gains on A 4 miles each hour, he will overtake him when he has gained the entire circumference; that is, A and B will be together at the end of every 5 hours. Again, since C gains on B 4 miles each hour, he will overtake him when he has gained the whole circumference; that is, B and C will be together at the end of every 5 hours. Consequently, they will all be together at the end of every 5 hours.

44. What is the discount of $175
cent.?

45. If a ship and its cargo is wor
cargo is worth 5 times as much as th
value of the cargo?

46. What is the difference between
times 7, and seven and one half times 6?

47. Three persons, A, B, and C, form
A furnishes $1000, B $600, and C $450;
6 months, C withdraws his capital, but no di
until the end of the year, when it is found
has gained $244.16. How is this gain
between the partners?

Ans. $\begin{cases} \text{A has } \frac{40}{73} \text{ of } \$244. \\ \text{B has } \frac{24}{73} \text{ of } \$244. \\ \text{C has } \frac{9}{73} \text{ of } \$244. \end{cases}$

48. Three persons, A, B, and C, engage
tain piece of wall for $244.16. While A
rods, B can build but 6, and C but $4\frac{1}{2}$. When
half completed, C ceases to labor upon it, and A
it. What part of the $244.16 ought each to receive

Had C continued through the whole work, it is e
that the money should have been divided between th
the ratio of the numbers 10, 6, and $4\frac{1}{2}$, or which is
same as the numbers 20, 12, and 9. So that A w
have $\frac{20}{41}$, B $\frac{12}{41}$, and C $\frac{9}{41}$ of the money.

But, since C continued until half the wall was finish
it is evident that for the first half they ought to receive
follows:

A ought to have $\frac{20}{41}$ of $\frac{1}{2}$ of $244.16=$59.55
B " $\frac{12}{41}$ of $\frac{1}{2}$ of $244.16=$35.73
C " $\frac{9}{41}$ of $\frac{1}{2}$ of $244.16=$26.80

$122.08

...nd B alone finished the second half of the ...one must share of $\frac{1}{2}$ of $244.16, in the ratio ...ers 10 and 6, or, which is the same, as the ...nd 3. So that

...ght to have $\frac{5}{8}$ of $\frac{1}{2}$ of $\$244.16=\$76.30$
" $\frac{3}{8}$ of $\frac{1}{2}$ of $\$244.16=\$45.78$
$122.08

...ng these results, we find that

A	ought to have	$135.85
B	"	$ 81.51
C	"	$ 26.80
	Proof	$244.16

...and B together can build a wall in 4 days, A and ...together build it in 5 days, B and C can together ...t in 6 days. What time would it require for all ...r to accomplish it?

...and B can in one day build $\frac{1}{4}$ of it$=\frac{15}{60}$ of it.
...and C " $\frac{1}{5}$ of it$=\frac{12}{60}$ of it.
and C " $\frac{1}{6}$ of it$=\frac{10}{60}$ of it.

...e sum of these fractions, $\frac{15}{60}+\frac{12}{60}+\frac{10}{60}=\frac{37}{60}$, is evi...y twice the fractional part accomplished by all in one ... Hence, they all would in one day accomplish $\frac{1}{2}$ of ...$=\frac{37}{120}$; consequently, in $\frac{120}{37}=3\frac{9}{37}$ days, they would ...sh it.

50. A note of $10000 given Jan. 1st, 1840, has received ...e following endorsements: Jan. 1st, 1841, endorsed ...2952.28, Jan. 1st, 1842, endorsed $2952.28, Jan. 1st, ...843, endorsed $2952.28. How much remained due Jan. ...st, 1844, interest being computed at 7 per cent.?

Ans. There was due $2952.28.

51. Two hunters, A and B, kill a deer, whose weight they are desirous of knowing. For this purpose, they rest a stick across the limb of a tree; then suspending the deer at the shorter extremity, they find that its weight is just counterpoised by the weight of A, who suspends himself by his hands at the other extremity. Without chang-

ing the point of support of the stick, they take the deer from the shorter extremity and suspend it at the longer extremity of the stick, when it was found to be exactly balanced by B's weight, when suspended at the shorter extremity of the stick. Now, supposing A to weigh 147 pounds, and B to weigh 192 pounds, what must have been the weight of the deer?

By the principle of the lever, we know that when different weights at its extremities balance each other, they are to each other inversely as the lengths of the arms to which they are attached. Hence, in the first experiment, we know that the weight of A is to the deer's weight, as the shorter arm is to the longer arm. In the second experiment, the deer's weight is to B's weight, as the shorter arm is to the longer arm. Consequently, A's weight is to the deer's weight, as the deer's weight is to B's weight; that is, the deer's weight is a mean proportional between A's weight and B's weight. Therefore, if we multiply the number of pounds which A weighed, by the number of pounds which B weighed, and extract the square root of the product, it will give the weight of the deer in pounds.

$$147 \times 192 = 28224.$$

And $\sqrt{28224} = 168$ the weight of the deer in pounds.

THE END.

KJ

HM